CONTRIBUTION TO MODELLING OF SLEEPER BED

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Abstract

Increasing demands for the quality of sleeper bed structure result from the fact that the trains with the vehicles of large axle weight achieve on multimodal railway tracks relatively high speeds. The design method for the sleeper bed construction currently in use in the Slovak Republic does not take into account all factors which can influence deformation and stress in its individual parts. This can have negative impact on the effective utilisation of individual parts of the sleeper bed. The article deals with the modelling of the sleeper bed structure which is simulated by a layered half-space with the layers of constant thickness.

Keywords: sleeper bed, stress, deformation modulus

1 Introduction

The demands on the track formation and particularly on the sleeper bed structure increase permanently nowadays since the speed of trains on multimodal railways reach 200km·h^{-1} and the axle weights of vehicles are of approx. 225 kN. The required road safety and the track formation reliability cannot be achieved then without increasing quality of railway construction. In present economic circumstances it is necessary to improve not only the work effectiveness but also to achieve the highest possible exploitation of construction materials. In this context effective design methods for railway construction are of the highest priority.
The design method for the sleeper bed construction currently used in the Slovak Republic [1] does not take into account all important factors which can influence deformation and strain and stress in a specific part of sleeper bed. This can have negative influence on effective use of individual parts of sleeper bed whose components have to be of a high quality since only this can guarantee required quality of the geometrical position of rails during their service life. This article deals with the modelling of sleeper bed structure which is in the calculations simulated by a layered half-space with the layers of constant thickness.

2 Basic assumptions of the design method

The basic requirement of the design method for the sleeper bed construction which is in use in the Slovak Republic is to know the values of deformation modulus and the thicknesses of individual layers in the sleeper bed. The method itself consists in the substitution of 2 layers of sleeper bed which can differ in their properties by the equivalent homogeneous half-space. This idea was proposed by Pokrovskij and it is based on the fact that if the same force is acting on two different elastic beams of the same width b, the stress on the half-space surface will be the same if the flexural rigidity of both beams are the same, i.e.,

$$E_0 I_0 = E_1 I_1 \quad (1)$$

where E_0, E_1 are the deformation moduli and I_0, I_1 the second moments of area of the beams.

The relationship between the heights of the beams of the same flexural rigidity is given by the equation:

$$h_e = h_1 \sqrt[3]{\frac{E_1}{E_0}} \tag{2}$$

The upper layer with thickness h_1 was substituted by the equivalent layer with thickness h_e with the properties of sleeper bed and its thickness is given by equation (2). This equation was modified by Ivanov for multilayered constructions:

$$h_e = h_1 \sqrt[2,5]{\frac{E_1}{E_0}} = h_1\ n \tag{3}$$

where,

$n^{2,5} = \frac{E_1}{E_0} = \frac{1}{k_1}$ is deformation characteristics of the system.

The method for the determination of equivalent deformation modulus of multilayered system was worked out by Ivanov in 1943 and it is known as DORNII method. Based on this method the formula for the equivalent deformation modulus E_e of multilayered system was derived [2], [3]:

$$E_e = \frac{E_1}{n^{2,5}\left[1 - \frac{2}{\pi}\left(1 - \frac{1}{n^{3,5}}\right)\ \text{arctg}\ k_2 n\right]} = E_1 k_3 \tag{4}$$

where $k_2 = \frac{h_1}{r_0}$, r_o – the radius of loaded plate (Fig. 2).

In practice E_e is determined from the graph drawn for the maximum layer thickness of 0.6m and the above mentioned constants k_1, k_2 and k_3 which is included in the standard [1].

3 Assessment of layered system

The layers of sleeper bed are assessed by the deformation resistance of the earth subgrade as well as the railway substructure. The smallest allowed values of static deformation modulus for individual parts of railway substructure are listed in Tables 1 and 2.

Table 1 Required values of static deformation modulus E_{sg} (Figure 1.) for the earth subgrade for different speed ranges

Speed Range	Speed V (km/h)	E_{sg} (MPa)
RP1	V ≤ 60	≥ 15
RP2	60 ‹ V ≤ 90	≥ 20 (≥ 15)*
RP3	90 ‹ V ≤ 120	≥ 30 (≥ 20)*
RP4	120 ‹ V ≤ 160	≥ 40 (≥ 30)*
RP5	160 ‹ V ≤ 200	≥ 50

* valid for tracks in use

Table 2 Required values of static deformation modulus E_{sb} (Figure 1.) for the railway substructure for different speed ranges

Speed Range	Speed V (km/h)	E_{sb} (MPa)
RP1	V ≤ 60	≥ 30
RP2	60 ‹ V ≤ 90	≥ 40 (≥ 30)*
RP3	90 ‹ V ≤ 120	≥ 50 (≥ 40)*
RP4	120 ‹ V ≤ 160	≥ 80 (≥ 50)*
RP5	160 ‹ V ≤ 200	≥ 100

* valid for tracks in use

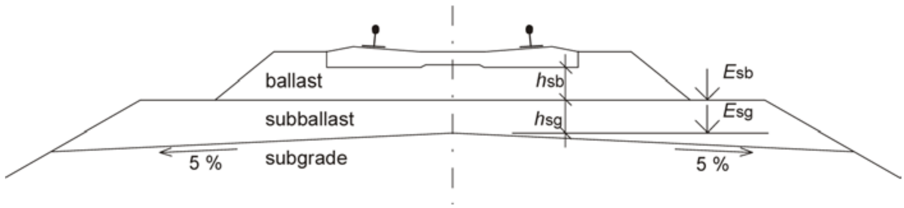

Figure 1 Requirements for deformation resistance of the sleeper bed structure

4 Numerical modelling of the sleeper bed structure

It is obvious that to solve the problem of stress and deformation in the sleeper bed structure is a 3-dimensional task from the mathematical and geometrical points of view. The stress and deformation in the structure is derived on assumption that the region of sleeper bed in shape of circular plate is under rotationally symmetric loading. In rotationally symmetric state the components of displacement as well as the components of stress and deformation depend only on two coordinates *r* a *z*, they do not depend on φ (Fig. 2).

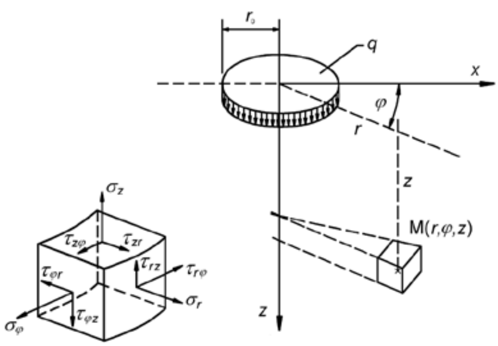

Figure 2 Tensor components and rotational symmetry of sleeper bed loading

The sleeper bed is considered to be an elastic body which consist of *n* homogeneous layers of constant thickness $^{(i)}h$ which lay on the $(n+1)$-th layer of a homogeneous half-space. Individual homogenous layers are made from elastic and isotropic materials characterized by $^{i)}E_o$ and $^{(i)}\nu$ [3],[4],[5]. Geometrical and physical characteristics of the layers can be seen in Fig. 3. The surface of the layered sleeper bed is under normal loading q_z homogeneously distributed over the area of circular plate of radius r_o.

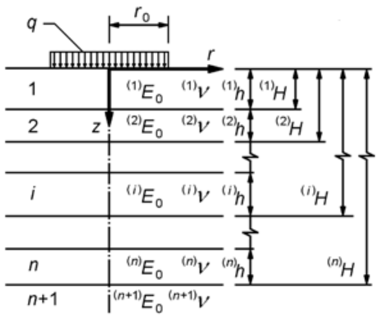

Figure 3 Scheme of the layered sleeper bed structure

The solution is based on the Boussinesq procedure, which on condition that the volume forces are constant, makes possible to transform Lame's and Beltrami's equations into biharmonic equations [4],[7]. Their solution is the following:

$$\sigma_r = \frac{\partial}{\partial z}\left(\nu\Delta F - \frac{\partial^2 F}{\partial r^2}\right)$$

$$\sigma_\varphi = \frac{\partial}{\partial z}\left(\nu\Delta F - \frac{1}{r}\frac{\partial F}{\partial r}\right)$$

$$\sigma_z = \frac{\partial}{\partial z}\left((2-\nu)\Delta F - \frac{\partial^2 F}{\partial z^2}\right) \tag{5}$$

$$\tau_{rz} = \frac{\partial}{\partial r}\left((1-\nu)\Delta F - \frac{\partial^2 F}{\partial z^2}\right)$$

$$u = -\frac{1+\nu}{E}\frac{\partial^2 F}{\partial r \partial z}$$

$$w = \frac{1+\nu}{E}\left(2(1-\nu)\Delta F - \frac{\partial^2 F}{\partial z^2}\right)$$

where $F(r,z)$ is the stress function which is the general solution to biharmonic equation:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2 F(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial F(r,z)}{\partial r} + \frac{\partial^2 F(r,z)}{\partial z^2}\right) = 0 \tag{6}$$

Biharmonic stress funcion $^{i)}F(r,z)$ which determines the stress state in i-th layer can be derived using Hankel's transformation [4], [7]. If the stress function is known then the stress and displacement of the i-th layer of sleeper bed can be calculated using equations (5). The number of unknown quantities has been reduced to $4n+2$. These quantities can be calculated taking into account two equations resulting from the boundary conditions for the surface $z = 0$:

1 $\quad {}^{(i)}\sigma_z(r,\ 0) = q, \quad \text{if} \quad r \le r_0$

$$ {}^{(i)}\sigma_z(r,\ 0) = 0, \quad \text{if} \quad r > r_0 \tag{7} $$

2 $$ {}^{(i)}\tau_{rz}(r,\ 0) = 0 $$

and other $4n$ equations for n contact surfaces $z = {}^{(i)}H$ of adjacent layers:

1 $$ {}^{(i)}\sigma_z\left(r, {}^{(i)}H\right) = {}^{(i+1)}\sigma_z\left(r, {}^{(i)}H\right) $$

2 $$ {}^{(i)}\tau_{rz}\left(r, {}^{(i)}H\right) = {}^{(i+1)}\tau_{rz}\left(r, {}^{(i)}H\right) \tag{8} $$

3 $$ {}^{(i)}w\left(r, {}^{(i)}H\right) = {}^{(i+1)}w\left(r, {}^{(i)}H\right) $$

4 $$ \left(1 - {}^{(i,i+1)}\tilde{y}\right) {}^{(i)}u\left(r, {}^{(i)}H\right) = \left(1 - {}^{(i,i+1)}\tilde{y}\right) {}^{(i+1)}u\left(r, {}^{(i)}H\right) + \frac{{}^{(i,i+1)}\tilde{y}}{2\ {}^{(i+1)}G} {}^{(i+1)}\tau_{rz}\left(z, {}^{(i)}H\right) $$

The quantity ${}^{(i,i+1)}\tilde{\gamma}$ characterizes interaction between layers *i* and *i*+1 at the contact surfaces $z = {}^{(i)}H$.

5 Numerical calculations

Illustrative results of the calculation of sleeper bed structure (shown in Figure 3,) were obtained on assumption that the sleeper bed is divided into active and passive layers and taking into account the ${}^{(i,i+1)}\tilde{\gamma}$ values characterizing layers interactions shown in Figure 4. The case when there is no shear stress at the contact surfaces of sleeper bed – base layer and base layer – earth subgrade is shown in Figure 5.

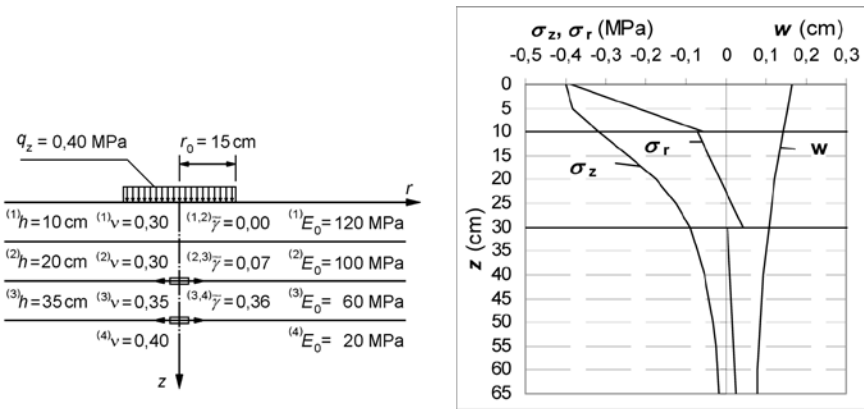

Figure 4 The course of studied quantities in the sleeper bed with interacting layers

From the analysis of stresses in sleeper bed results that under loading and free sliding of layers the local bending of the track bed takes place. This results in the considerable increase of radial tensile stress on the bottom surface of this layer. The bend of this type of sleeper bed also considerably increases (displacement w in z direction).

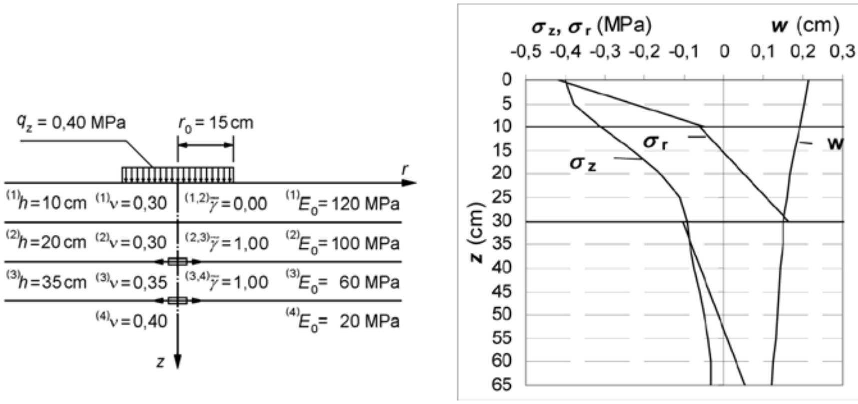

Figure 5 The course of studied quantities with no shear stress at the contact surfaces sleeper bed – base layer and base layer – earth subgrade

6 Conclusion

The presented method for the calculation of stress and displacement in the layered structure of the sleeper bed takes into account more properties of the sleeper bed materials. It enables more effective design of sleeper bed and therefore it could be considered as a good substitute for so far used method. The method takes into account the interaction of individual layers which is of great importance for the calculation of their bend loading and extent of deformation. However, prior to the implementation of this method it is necessary to define the criteria for the assessment of the bearing capacity of the sleeper bed structure.

Acknowledgements

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References

[1] TNŽ 73 6312 Navrhovanie konštrukčných vrstiev podvalového podložia. 2005

[2] MANDULA ,J.: Modelovanie konštrukcie podvalového podložia s geosyntetickou výstužnou vrstvou, In: Zb. z konferencie s medzinárodnou účasťou: Rozvoj dopravy v Košiciach a východoslovenskom regióne, Košice, 2002, ISBN 80-232-0209, s. 265 – 268

[3] MANDULA,J.: Modelovanie geosyntetickej vrstvy v konštrukcii podvalového podložia, In: Zb. z 2. seminára s medzinárodnou účasťou: Geosyntetika v dopravnom staviteľstve, Žilina, 2003, ISBN 80-8070-033-8, s. 43 – 46

[4] MAINA, J., MATSUI, K.: Elastic multi-layered analysis using DE-integration. Publ. Research Institute for Mathematical Sciences, Kyoto University 41 (2005), 853–867

[5] ROSE, J., SU, B., LONG, W.: (2002) Kentrack: a railway trackbed structural design and analysis program. This paper is a condensed version of a THESIS submitted by Bei Su in partial fulfillment of the requirements for the degree of master of science in Civil Engineering at the University of Kentucky

[6] MARTINČEK, G.: Dynamics of pavement structures. E& FN SPON, London 1994

[7] NOVOTNÝ, B, HANUŠKA, A.: Teória vrstevnatého polpriestoru. VEDA Bratislava, 1983