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DETERMINATION OF TORSIONAL RESISTANCE OF RAIL FASTENINGS AND THEIR EFFECT ON STABILITY OF THE RAILWAY TRACKS

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Abstract

In calculations of the stability of railway tracks, the effect of torsional resistance of rail fastenings can be taken into account by two principal methods. One of them is the effect of a single fastening that can be determined according to the procedures of the standard EN 13146-2:2012, and the other method takes into consideration the replacement moment of inertia of the entire track panel. In Hungary two analytical methods are used to calculate the stability of railway tracks against buckling. One of them is the Nemesdy-method that applies the torsional resistance of a single fastening determined by the standard. The other one is Meier's method, that uses the replacement moment of inertia of the total track panel. This paper gives a brief overview on the procedures of the standard to measure the torsional resistance of a single fastening and presents a FEM model on how to compute the replacement moment of inertia of the track panel. Calculations have also been performed on the stability of curved tracks to determine the minimum curve radius by both methods and conclusions have been drawn on the percentage of the contribution of torsional stiffness to the total stability of the railway tracks. A wide variety of finite-element software are available; the modelling of track buckling can be found on the basis of the concepts presented here.

Keywords: torsional resistance, rail fastening, replacement moment of inertia, buckling, stability of railway tracks

1 Introduction

In Hungary two most important analytical methods are used for the calculation of stability of tracks against buckling:

- the Nemesdy-method that uses the torsional resistance of the rail fastenings and considers the equilibrium of forces
- the Meier's method that uses the replacement moment of inertia of the track panel and calculates with the equation of the works.

2 The Nemesdy method and the torsional resistance

2.1 Principal of the Nemesdy method [1]

The railway tracks are never straight in reality but always have a series of eccentricities and misalignments which are sufficiently small when properly maintained. Three factors inhibit the lateral displacement and buckling:

- bending stiffness of the two rails
- torsional resistance of rail fastenings
- lateral ballast resistance.

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In state of equilibrium the sum of the three above forces is equal to the critical normal force of in the rails causing buckling of the track. The critical load of a beam fixed at both ends according to the Euler theorem:

$$F_{A,\ Euler} = 4\pi^2 \frac{E \cdot I}{\ell^2} \tag{1}$$

Using the differential equation of an elastic beam that takes into account the stiffness of the rail section and the frame rigidity effect characterized by the torsional resistance constant, the value of the critical compressive force is:

$$F_{A2} = 40\frac{E \cdot I}{\ell^2} + \frac{2r}{k} \tag{2}$$

The lateral resistance of the ballast is interpreted as consisting of approximate parabolas of the misalignment shape. In case of a straight track with misalignment of type „A", the critical force causing buckling:

$$F_{crA} = 40\frac{E \cdot I}{\ell^2} + \frac{2r}{k} + \frac{\ell^2}{10f}q \tag{3}$$

The formulas for curved tracks were developed by modifying the calculation expressions developed for straight tracks. In the case of curved geometry with radius R, only a residual part of the ballast lateral resistance equal to q - F/R remains effective for maintaining stability. The critical force causing buckling:

$$F_{cr,R} = \frac{40 \cdot \frac{EI}{\ell^2} + \frac{2 \cdot r}{k} + \frac{\ell^2}{10 \cdot f} \cdot q}{1 + \frac{\ell^2}{10 \cdot f \cdot R}} \tag{4}$$

In case of curved tracks, the minimum curve radius can be obtained from the following equation, smaller curve radii will result in buckling of the track:

$$R_{cr,min} = F_{rail} / \left\{ \left[q - \frac{\left(F_{rail} - 2 \cdot r / k\right)^2}{16 \cdot EI} \right] \cdot f \right\} \tag{5}$$

In the above equations, E is the elasticity modulus of the rail, I is the moment of inertia of the two rails, l is the length of misalignment, r is the torsional resistance of the rail fastenings, k is the sleeper spacing, f is the height of misalignment, q is the lateral ballast resistance and R is the radius of the curve.

2.2 Determination of torsional resistance of a single rail fastening according to the standard EN 13146-2:2012 [2]

The standard EN 13146-2:2012 [2] specifies a simple laboratory test procedure to measure the torsional resistance of rail fastenings. A brief extract of the procedure is the following: An increasing load is applied to the rail foot, and the moment of load and the displacement relative to the sleeper are continuously measured. When the rail has displaced by at least 1, 5 degrees, the load is removed, [...] and then the load is applied to the opposite side of the rail and the same loading cycle is repeated. The moment to cause an angular displacement of 1° is then determined. The value obtained can be used in track stability calculations. The standard does not specify any algorithms or calculation procedures on how the measured values can be used in further calculations.

Possible results of first and return tests carried out on a W14 fastening are shown in figure 1. The result of the first test is 0.506 Nm, and that of the return test is 0.502 kNm. On average the torsional resistance is 0.504 kNm/degree that is equivalent to 28.877 kNm/radian.

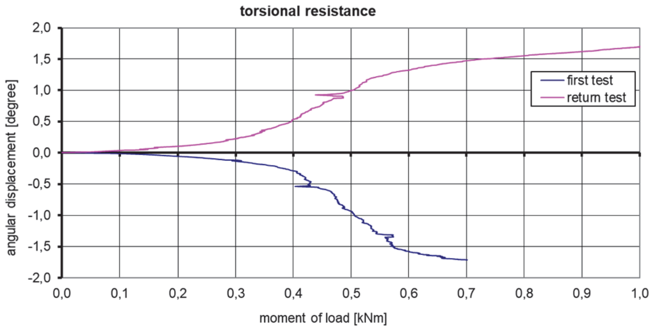

Figure 1 Torsional resistance test results on W14 fastening

3 The Meier method and the replacement moment of inertia of a track panel

3.1 The Meier method [3]

Meier published a calculation procedure with the energy method to analyze the stability of the continuously welded rail track against buckling for straight and curved tracks. In state of equilibrium the work of the external forces is equal to the work of the internal forces:

$$W_F = W_E + W_q \tag{6}$$

where

- W_F - is the work done by the normal compression force in the rail resulting from increasing temperature
- W_E - is the work of the bending stiffness of the track panel including the rails, fastenings and the sleepers that is the work of the Euler force
- W_q - is the work carried out by the lateral ballast resistance force.

The critical increase of temperature ΔT_{cr} of the rail is that will cause buckling of the track in case curved track:

$$\Delta T_{cr} = -\frac{8\cdot I_{track}}{\alpha\cdot A_{2r}\cdot R\cdot f} + \sqrt{\left(\frac{8\cdot I_{track}}{\alpha\cdot A_{2r}\cdot R\cdot f}\right)^2 + \frac{16\cdot I_{track}\cdot q}{\alpha^2\cdot A_{2r}\cdot E\cdot f}} \tag{7}$$

In Meier's theorem the I_{track} is the replacement moment of inertia of the total track panel consisting of the two rails, the rail fastenings and the sleepers. A_{2r} is the sum of the cross sectional area of the two rails, *a* is coefficient of linear thermal expansion, R is the radius of the curve, f is the height of misalignment, q is the lateral ballast resistance and E is the elasticity modulus of the rail.

3.2 Determination of the replacement moment of inertia of a track panel by laboratory tests

The replacement moment of inertia of a track panel consisting of two rail sections, sleepers and rail fastenings can be determined by laying the track panel on two quasi-frictionless rollers, the panel is supported laterally at the two ends and is loaded at midspan by a concentrated load in the horizontal plane. From the deflection of a two-support beam loaded by concentrated load at midspan, the replacement moment of inertia of the track panel is:

$$I_{track} = \frac{F \cdot l^3}{48 \cdot E \cdot y} \tag{8}$$

As a result, the replacement moment of inertia of a track panel consisting of two rail sections of 54E3, concrete sleepers of B70 and rail fastenings W14 is $I_{track} = 14,33 \cdot 10^6$ mm^4 and it is twice higher than the moment of inertia of two 54E3 rail sections – $7,096 \cdot 10^6$ mm^4 – due to the torsional resistance of the rail fastenings. A similar track grid with two 60E1 rails has a replacement moment of inertia of $22,00 \cdot 10^6$ mm^4, whereas two 60E1 rails have a moment of inertia of $10,246 \cdot 10^6$ mm^4 [4, 5, 6]. The frame stiffness and the substitutional inertia is analyzed by Major et al [7]. Schmid investigates the influence of lateral wheelset force on track buckling [8].

4 Determination of the replacement moment of inertia of a track panel with FEM software

This section provides methods on how to determine the replacement moment of inertia of a regular track panel and also for that of a special track panel.

4.1 Cross-sleepered track panel

Based on the tests described in the previous section, a model of a track panel with 11 B70 sleepers, two 54E3 rails and rail fastenings was built up with FEM software Axis VM. The sleeper spacing was 600 mm. The sleepers were defined by their cross-sectional area under the railseat over a length of 1100 mm measured from the end of the sleepers, and the middle part of the sleeper over a length of 400 mm was defined by its middle cross-sectional area. The strength of the concrete is C50/60. The rail fastenings were replaced by beams whose material properties were chosen so the replacement moment of inertia of the FEM model coincides with that measured in literature [5] that is $I_{track} = 14.33 \cdot 10^6$ mm^4. The material properties of the beams substituting the rail fastenings are:

• length l:	183 [mm]
• cross-sectional area A:	150×150 = 22500 [mm^2]
• elasticity modulus E:	920 [N/mm^2]
• Poisson ratio v:	0, 35 [-].

In a project in Hungary, the stability of tracks with rail sections of MÁV 48 and 54E1 and also with Hungarian concrete sleeper types of LM were modelled. In the FEM model, the cross-sections of the sleepers were replaced by the cross-sections of LM sleepers under the rail seat, over a length of 910 mm measured from the end of the sleepers whose total length is 2,420 mm. The middle part of the sleepers over a length of 600 mm was defined by their middle cross-sectional area. The rails were substituted by profiles of the MÁV 48 and the 54E1 sections. The material properties of the beams representing the rail fastenings were preserved.

The deformed shape of the track panel with 54E1 rails supported in the horizontal plane at the two end-sleepers and loaded horizontally at midspan by 20 kN is illustrated in figure 2. The horizontal lateral displacement of the rail at midspan is e_y = 24.990 mm with 54E1 rails, and e_y = 29.666 mm with MÁV 48 rails. The replacement moment of inertia of the track panel with LM sleepers is:

$$I_{track} = 16,751\cdot 106\ mm^4 \text{ with 54E1 rails} \tag{9}$$

$$I_{track} = 14,110\cdot 106\ mm^4 \text{ with MÁV 48 rails} \tag{10}$$

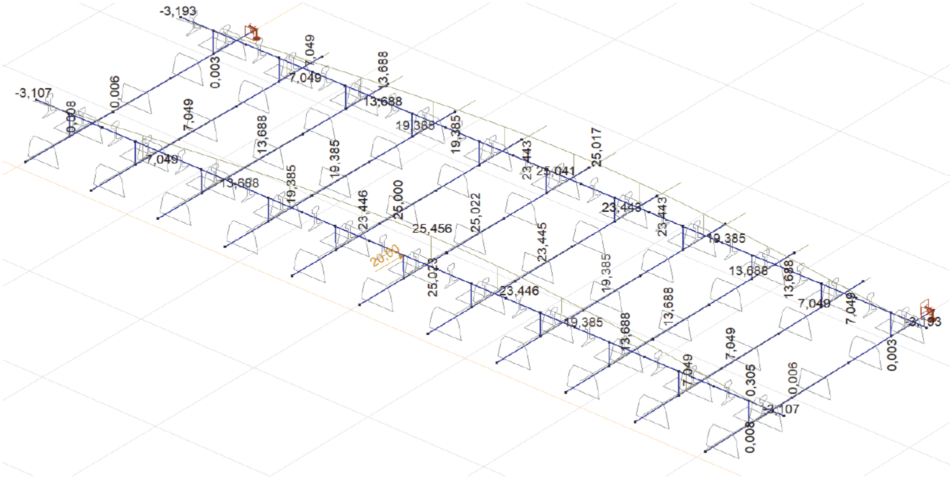

Figure 2 Deformed shape of the track panel with LM sleepers and 54E1 rails

4.2 Replacement moment of inertia of the track of the cogwheel railway

With the method presented in the previous section, the replacement moment of inertia of any special form of tracks can be determined. An example is presented on the track of the Budapest Cogwheel Railway. This has running rails of MÁV48 sections and sleepers of LM type. The cogwheel is fastened in the center line of the track with special Skl14 fastenings. Figure 3 illustrates a photo of it. A similar FEM model has been built up for the track of the cogwheel railway as in the previous section. The rail has been loaded by 20 kN of horizontal lateral load at midspan, and the model was supported at the two end-sleepers. If all rail fastenings are considered to be tight, and all rails are considered to be continuously welded then the replacement moment of inertia of this track is $I_{track} = 25.786 \cdot 10^6\ mm^4$. A sensitivity analysis can be performed on how tight the fastenings are. By altering the torsional stiffness of the beams representing the fastenings, the replacement moment of inertia of the track with any kind of fastening can be determined. The rail joints of conventional rails will also influence the replacement moment of inertia of the track. The FEM modelling introduced in the previous section is capable to model any kind of sections of cogwheels and/or running rails and fastenings, however, due to their extent, it will be a topic of another paper.

Figure 3 A photo of the track of Budapest Cogwheel Railway

5 Calculations of track stability

According to the instructions of MÁV Zrt D.12.H of Hungarian Railways, that is the instructions of construction and maintenance of CWR tracks, the nominal value of the neutral temperature of the rail is 23°C and the neutral temperature zone is °C, therefore its lowest value is 15°C. [9] The instructions suggest that the maximum rail temperature should be considered to be 60°C in summer due to direct sunlight, however in the last 20 years there have been summers when the rail temperature exceeded 60°C. A sensitivity analyses has been carried out for the case when the neutral temperature decreases down to 10°C during the operations and maintenance works, therefore a temperature increase of 55°C has been taken into account in the calculation of the critical curve radii in respect of buckling.

In the calculations, the lateral resistance of the ballast was assumed to be 5, 10, and 15 N/mm. The lateral resistance in freshly laid good ballast or in contaminated poor-quality ballast is 5–6 N/mm or less, in good-quality compacted but unconsolidated ballast it is 8–10 N/mm, and in good quality consolidated subgrade it can be 15–18 N/mm. Additional structural solutions, such as widening and raising the ballast shoulder, using sleeper anchors, ballast bonding, or constructing supporting beams, can further increase the lateral resistance of the ballast bed [4, 9]. The heights of the misalignment errors were chosen to be f = 5, 10, 20 and 30 mm and examined how the error height affects the value of the critical curve radius.

Table 1 Minimum curve radii [m] according to Meier’s method

Height of misalignment [mm]	Ballast resistance [N/mm]					
	5	10	15	5	10	15
	with moment of inertia of the entire track panel			with sum of moment of inertia of the two rails		
5	404.8	195.9	129.2	433.7	202.4	132.0
10	433.7	202.4	132.0	505.8	216.8	138.0
20	506.0	216.9	138.0	758.1	252.9	151.8
30	607.2	233.5	144.6	1512.6	303.4	168.6

Table 2 Minimum curve radii [m] according to Nemesdy method

Height of misalignment [mm]	Ballast resistance [N/mm]					
	5	10	15	5	10	15
	with measured torsional resistance			neglecting torsional resistance		
5	427.6 (0.89%)	201.1 (0.46%)	131.4 (0.31%)	433.7	202.4	132.0
10	489.7 (1, 65%)	213.8 (0, 89%)	136.8 (0.61%)	505.8	216.8	138.0
20	690.0 (2, 90%)	244.9 (1.65%)	148.8 (1.16%)	758.1	252.9	151.8
30	1167 (3.88%)	286.4 (2.32%)	163.2 (1.65%)	1512	303.4	168.6

Table 1 indicates the critical curve radii in respect of buckling of track with rail section of 54E1 with temperature increase of 55°C, according to Meier's method. A sensitivity analysis has been carried out, the calculations have been performed with the assumption of the replacement moment of inertia of the track panel with tight rail fastenings introduced in section 4.1, and with the assumption that the torsional resistance of the rail fastenings is neglected, in the latter case the moment of inertia of the track panel is equal to the sum of the two rails relative to their vertical symmetry axis.

A similar procedure has been carried out with Nemesdy method, the critical curve radii were computed with the measured torsional resistance presented in section 2.2 and also with considering the torsional resistance to be zero (table 2). Table 2 also contains in brackets the percentage of the total force resisting against buckling that is contributed by the torsional resistance of the rail fastenings, assuming a misalignment length of 10 m. As it is indicated, this is a low percentage in case of a ballast bed with good quality. The torsional resistance of the fastenings have a higher percentage of contribution to stability only in case of a ballast with poor quality with low resistance and in case the track has high inherent geometrical misalignments.

6 Conclusion

This article has given an overview of two laboratory test methods, one of them is to measure the torsional resistance of a single rail fastening and the other one is to determine the replacement moment of inertia of an entire track panel and two analytical calculation procedures have been discussed on how their results can be applied for calculations of track stability. The paper introduces a FEM modelling on how to determine the replacement moment of inertia of a track. Any kind of rail fastening, sleeper type, sleeper material, rail sections, special rails can be modelled by utilizing this procedure. The calculations highlight that torsional resistance has less importance in the contribution to the stability of the track. It can be concluded that the two analytical methods will result in the same minimum curve radius values (tables 1 and 2) if the torsional resistance of the fastenings is neglected. The Nemesdy method, that is based on the torsional resistance of a single fastening, will result in somewhat higher minimum curve radius values that will support safety.

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