



MACHINE LEARNING METHODOLOGY TO PREDICT THREE-DIMENSIONAL WHEEL-RAIL CONTACT FORCES IN HYBRID VEHICLE-STRUCTURE INTERACTION ALGORITHM

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Abstract

Wheel-rail contact modelling is one of the most important aspects of vehicle-structure interaction algorithms as it provides the coupling between the vehicle and the track and heavily influences the behaviors of the contact forces and vehicle dynamic responses. The wheel and rail geometries and materials dictate the contact behaviors that can be modelled using complex and sophisticated specialized algorithms. Realistic modelling of contact forces are essential to the assessment of train running safety, requiring highly non-linear algorithms to capture the dynamic behaviors of the vehicles accurately, albeit at a significant computational cost and implementation complexity. Due to these drawbacks, this paper introduces a machine learning approach to estimate wheel-rail contact forces and dynamic parameters within a non-linear three-dimensional vehicle-structure interaction framework. The proposed approach is aimed at improving the computational efficiency of these algorithms by replacing conventional contact formulations by an alternative machine learning algorithm. A three-dimensional nonlinear vehicle-structure interaction model is employed to train and validate the hybrid algorithm, with neural networks trained on a numerically generated dataset of dynamic parameters of the train during the crossing of bridge encompassing diverse track irregularity profiles and a range of train speeds, covering the whole operational and conventional safety investigations. The dataset captures the complex nonlinearities of the wheel-rail contact as well as the behaviors of the structure, enabling the model to learn the full spatial dynamics of the vehicle-structure interaction. Numerical comparisons between the hybrid and conventional algorithms are conducted to assess the accuracy of the proposed method in reproducing system responses and contact forces, as well as a discussion on its computational performance. Results indicate that the machine learning models effectively reproduces the nonlinear contact behaviors with high fidelity for all contact parameters. It is also demonstrated that the hybrid algorithm can obtain accurate results in the non-linear dynamic solution. This framework highlights the potential for a robust and scalable machine learning approach for efficient three-dimensional vehicle-structure interaction and train running safety analysis, supporting advanced studies in structural dynamics, vibration control, and train running safety under realistic contact conditions.

Keywords: wheel-rail contact, vehicle-structure interaction, machine learning

1 Introduction

Train running safety on bridges is a critical issue in railway engineering, as dynamic interactions between vehicles and structures can significantly affect operational performance, structural integrity, and overall safety.

The evaluation of vehicle-structure interaction problems remains challenging, since commercial train-track interaction software typically provides limited representation of bridge structures, while general-purpose finite element software often lacks dedicated train-track interaction capabilities. As a result, most research in this field relies on numerical methods specifically developed for vehicle-structure interaction analysis. Train-track-bridge interaction models are traditionally formulated by enforcing compatibility conditions at the wheel-rail contact points, using either uncoupled approaches, where vehicle and structure models are solved separately, or coupled formulations, in which system matrices are solved simultaneously [1, 2]. In three-dimensional frameworks, the modelling of wheel-rail contact becomes considerably more complex due to geometric nonlinearities, non-conservative forces, and coupled vertical and lateral contact dynamics. The solution of the geometric contact problem, normal and tangential contact formulations [3] lead to high computational costs and increased modelling complexity, motivating the development of alternative approaches that preserve accuracy while improving numerical efficiency. Recent advances in machine learning have demonstrated the potential of data-driven models to complement or replace traditional engineering formulations, particularly for problems governed by strong nonlinearities [4]. Several studies have explored the use of machine learning techniques to predict wheel-rail contact forces and assess vehicle safety, including the prediction of complete contact force time histories [5]. Hybrid approaches employing surrogate models for wheel-rail contact have also been proposed to enhance the efficiency of conventional numerical algorithms [6]. These developments highlight the suitability of machine learning methods for capturing the complex behavior of wheel-rail interactions while reducing computational demands.

In this paper, a hybrid machine learning-based approach for nonlinear three-dimensional vehicle-structure interaction analysis is presented. The proposed framework employs machine learning models that use vehicle and structure information as inputs to reproduce three-dimensional wheel-rail contact behavior within a vehicle-structure interaction framework, including contact forces and associated kinematic quantities while the global dynamic problem is solved using conventional time integration methods. Section 2 describes the adopted vehicle-track-bridge interaction model, Section 3 presents the machine learning methodology, Section 4 details the dataset generation and hybrid algorithm implementation, and Section 5 discusses the numerical results.

2 Vehicle-structure interaction models

The methodology employed to solve the train-track-bridge interaction problem in this study is based on a three-dimensional nonlinear analysis considering a realistic wheel-rail contact model that accounts for both vertical and lateral interaction effects. The dynamic interaction solution method used is the direct method [7, 8], based on the Lagrange multiplier method, which creates a coupled global matrix of train-track and vehicle systems. The coupling between vehicle and track is represented through additional constraint equations that establish the displacement relations in the contact points, forming a single system of equations described in equation 1, where \bar{K} is the effect stiffness matrix, and \bar{D} and \bar{H} are the transformation matrices that relate the contact forces with nodal forces and the displacements in the structure, and $\Delta \mathbf{X}^{i+1}$ and $\Delta \mathbf{a}_F^{i+1}$ are the incremental nodal displacements and contact forces, $\bar{\mathbf{r}}$ is the residual force vector and $\bar{\mathbf{r}}$ is the vector with irregularities on the contact points.

$$\begin{bmatrix} \bar{K} & \bar{D} \\ \bar{H} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{a}_F^{i+1} \\ \Delta \mathbf{X}^{i+1} \end{bmatrix} = \begin{bmatrix} \Psi(\mathbf{a}^{t+\Delta t, i}, \mathbf{X}^{t+\Delta t, i}) \\ \bar{\mathbf{r}} \end{bmatrix} \quad (1)$$

The vehicle is modelled using a three-dimensional multibody dynamic model representative of a high-speed train Velaro S103, accounting for the essential inertial and suspension characteristics governing its dynamic response, as illustrated in figure 1a with properties adopted according to [9]. The bridge corresponds to a simply supported 30 meter long slab bridge with ballastless track with properties given in [10]. The structure is modelled using a three-dimensional finite element formulation represented by frame elements and discrete springs and dampers capturing the track support conditions, as shown in figure 1b. Track irregularities were generated using the spectral representation method [11] based on the PSD functions of the German spectra [12] for low, medium and high interference levels.

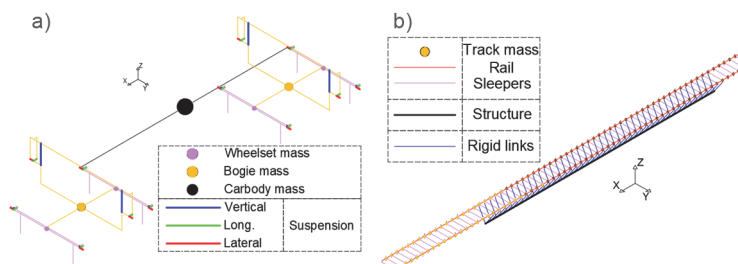


Figure 1 (a) Vehicle multibody and (b) bridge finite element models

3 Machine learning algorithms

The wheel-rail contact behavior is modelled using a feedforward neural network, selected for its robustness, ease of implementation, and ability to approximate strongly nonlinear relationships. Although feedforward architectures do not explicitly account for temporal dependencies, time-dependent effects are incorporated by augmenting the input space with response quantities from previous time steps, allowing the network to implicitly capture the system’s dynamic evolution. The adopted model consists of multiple fully connected hidden layers with nonlinear activation functions, providing sufficient expressive capacity to represent the complex mapping between the system state and the resulting contact behaviour while maintaining a compact and computationally efficient architecture, as illustrated generically in figure 2.

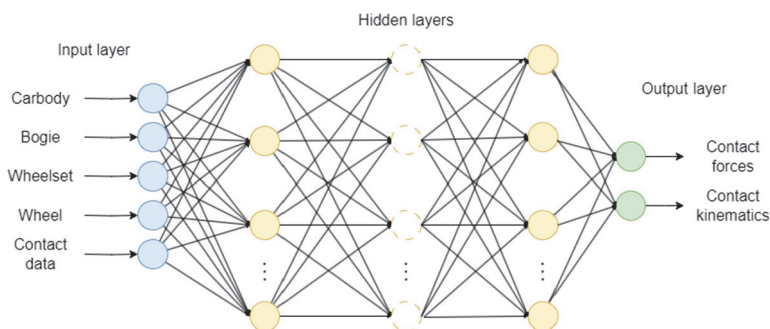


Figure 2 General architecture of the network

The network inputs investigated were the various dynamic responses of the vehicle at the previous time step, comprising all translational and rotational degrees of freedom of the wheel, wheelset, bogie, and carbody nodes, as well as contact translational components of the auxiliary contact node developed in the wheel-rail contact formulation [4]. These parameters capture the dynamic behavior of the vehicle and, indirectly, the stiffness characteristics of the structure by using data of the contact node, making it suitable for the vehicle-structure interaction methodology. The network outputs correspond to the full set of wheel-rail contact data at the current time step, including contact forces and the kinematics (auxiliary contact node's displacements, velocities and accelerations) which are part of the time integration solution of any dynamic problem.

4 Methodology

The proposed methodology combines a three-dimensional vehicle-structure interaction model with a machine learning-based wheel-rail contact formulation to form a hybrid dynamic analysis framework. The approach consists of two main stages: (i) generation of a dataset from numerical train-track-bridge interaction simulations exclusively during the crossing of the bridge for training and optimization of a neural network, and (ii) integration of the trained model into the nonlinear interaction algorithm, replacing the conventional contact solver.

4.1 Dataset generation and training strategy

The dataset used to train the machine learning model was generated from dynamic analyses performed with the conventional three-dimensional vehicle-structure interaction model. Simulations were conducted considering different train speeds and multiple track irregularity profiles, ensuring that the dataset captures a representative range of operating conditions and dynamic responses. From each simulation, time histories of relevant vehicle responses and wheel-rail contact quantities were extracted, forming the basis for supervised learning. A feature selection procedure based on permutation feature importance was first applied to the full set of candidate variables to identify the most influential input quantities governing the wheel-rail contact behavior, reducing redundancy while preserving the essential nonlinear dynamics of the interaction. Following feature selection, the neural network architecture, training hyperparameters, and temporal input window size were optimized to achieve an appropriate balance between prediction accuracy and numerical robustness.

4.2 Implementation of the hybrid algorithm

Once trained, the neural network was integrated into the vehicle-structure interaction framework to form a hybrid dynamic analysis algorithm. In this formulation, the conventional nonlinear wheel-rail contact model is replaced by the machine learning model as described in figure 3, which predicts the three-dimensional contact forces and associated kinematic quantities based on the dynamic responses of the vehicle and contact point at each time step. The hybrid model is embedded directly within the time-stepping solution procedure of the interaction algorithm, ensuring full coupling between the vehicle, track, and bridge subsystems. Since the machine learning model relies on response information from previous time steps, a limited number of initial time steps t_{AI} are computed using the conventional contact formulation to initialize the temporal inputs and avoid numerical disturbances commonly observed at the beginning of dynamic simulations. After this initial phase, the machine learning-based contact model is activated and used for the remainder of the analysis.

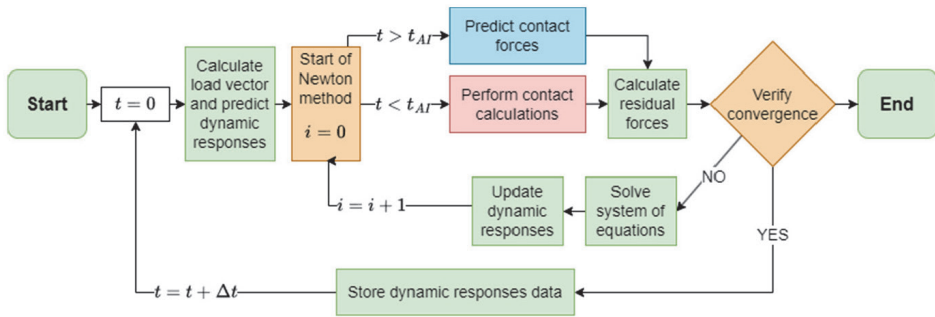


Figure 3 Simplified flowchart of the hybrid algorithm

5 Results

5.1 Network architecture and accuracy

Four separate feedforward neural networks with an augmented temporal input structure were adopted to model the three-dimensional wheel-rail contact behavior. After the feature selection and hyperparameter optimization procedures, the final network architectures are summarized in table 1. The selected configuration represents a compromise between model complexity and predictive capability, ensuring numerical robustness when integrated into the dynamic interaction algorithm.

Table 1 Network architectures

Outputs	Number of inputs	Number of layers	Units per layer
Contact forces	$43 + 20 \cdot 12$	7	512
Contact accelerations	$43 + 20 \cdot 9$	4	2024
Contact velocities	$43 + 20 \cdot 9$	4	512
Contact displacements	$43 + 20 \cdot 9$	4	512

Following training, the predictive accuracy of the neural network was evaluated, and the resulting error metrics are summarized in table 2. Absolute accuracy is quantified using the mean absolute error (MAE) and root mean square error (RMSE). In addition, normalized, dimensionless metrics are considered to enable a relative assessment of prediction accuracy, particularly for near zero-mean dynamic responses, including the peak-normalized mean absolute error (PNMAE), the mean absolute normalized error (MANE), and the normalized root mean square error (NRMSE).

The results indicate that the trained network is able to reproduce the wheel-rail contact quantities with high accuracy. The low error levels obtained demonstrate the capability of the proposed model to capture the strongly nonlinear relationships governing three-dimensional contact behavior, including the coupled vertical and lateral effects inherent to the interaction problem.

Table 2 Network performances on the test set

Outputs		MAE [SI]	RMSE [SI]	PNMAE [%]	MANE [%]	NRMSE [%]
Force [N]	X	5.77	7.93	0.168	0.731	0.817
	Y	12.99	18.63	0.179	0.844	0.955
	Z	16.23	22.50	0.018	0.021	0.029
Accl. [m/s ²]	X	0.007	0.011	0.602	3.966	4.680
	Y	0.043	0.064	0.538	2.124	2.532
	Z	0.037	0.051	0.347	1.426	1.624
Veloc. [m/s]	X	8.51E-06	1.31E-05	0.265	1.187	1.471
	Y	7.37E-05	1.13E-04	0.111	0.500	0.592
	Z	4.78E-05	6.36E-05	0.048	0.223	0.234
Displ [m]	X	1.56E-08	2.35E-08	0.009	0.030	0.037
	Y	1.29E-07	2.11E-07	0.008	0.028	0.037
	Z	2.74E-07	3.85E-07	0.006	0.013	0.017

5.2 Hybrid algorithm

The trained neural network was subsequently integrated into the vehicle-structure interaction framework, replacing the conventional nonlinear wheel-rail contact formulation. The dynamic responses obtained from the resulting hybrid algorithm were then compared with those computed using the original contact model for identical simulation scenarios. For a limited number of time steps, the hybrid vehicle-structure interaction algorithm accurately predicts the three-dimensional wheel-rail contact quantities and reproduces the dynamic responses obtained with the conventional nonlinear contact formulation. During this initial phase, the Newton-Raphson iterative procedure converges satisfactorily, indicating that the predicted contact forces and kinematic quantities remain consistent with system equilibrium. As the simulation progresses, a gradual deterioration of the convergence behavior is observed. Although the contact predictions remain locally accurate, small discrepancies accumulate over successive time steps, eventually leading to failure of the nonlinear solver to converge. To assess the performance of the hybrid algorithm prior to divergence, the average accuracy of the predicted contact quantities and dynamic responses for the converged time steps is reported in table 3. These results confirm the accuracy of the proposed contact surrogate at the time-step level, while highlighting the need for further developments to improve long-term numerical stability.

Table 3 Average error of the contact parameters in the hybrid algorithm

Outputs		MAE [SI]	RMSE [SI]	PNMAE [%]	MANE [%]	NRMSE [%]
Force [N]	X	5.77	7.93	0.168	0.731	0.817
	Y	12.99	18.63	0.179	0.844	0.955
	Z	16.23	22.50	0.018	0.021	0.029
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6 Conclusion

This paper presented a machine learning-based hybrid approach for modelling three-dimensional wheel-rail contact forces within a nonlinear vehicle-structure interaction framework. A feedforward neural network with an augmented temporal input structure was trained using data generated from detailed numerical simulations and integrated into the dynamic interaction algorithm to replace the conventional contact formulation. The results obtained from the trained network demonstrate its ability to accurately reproduce the strongly nonlinear three-dimensional contact behaviors. Preliminary results from the hybrid dynamic analyses indicate that the proposed approach can accurately reproduce vehicle and bridge responses for a limited number of time steps, with close agreement observed between the hybrid and conventional solutions. However, the long-term stability of the nonlinear solution procedure is currently limited by convergence issues in the iterative solver as small prediction discrepancies accumulate over successive time steps. Ongoing work is focused on improving the robustness of the hybrid algorithm, including enhanced coupling strategies and convergence control mechanisms. Despite these challenges, the present study establishes a solid foundation for the development of efficient and reliable machine learning-assisted vehicle-structure interaction models for advanced train running safety and structural dynamics assessments.

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