



## BUCKLING, QUANTS AND CONTINUITY: NEW PARADIGMS IN TRAFFIC FLOW

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### Abstract

Traffic is a union of flow regimes bound and defined by externalities such as roadway geometry, traffic control, interruptions, and climate conditions, as well as internalities such as driver behaviour, pedestrian behaviour and flow heterogeneity. The density along a roadway that manifests the speed of travel and volume of flow is a parameter of great interest, the control and monitoring of which is a direct determinant of traffic flow conditions. Traffic flow within and across roadway networks based on a hierarchy of accessibility and mobility is an area of study that can benefit from interdisciplinary research. In support of the many mathematical theories and empirically supported approaches developed to understand traffic flow, this study provides insight and a discussion concerning the application of mechanical theories to traffic flow. Classical mechanics through buckling theory and quantum mechanics through its discontinuous understanding of the nature of matter can address the issue of traffic flow. This study will revisit some of the contemporary questions that challenge traffic engineers and propose and raise hypothesis based on new approaches rooted in physics.

*Keywords: density, speed, flow rate, free flow, forced flow, headway*

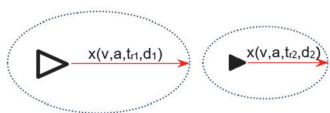
### 1 Introduction

Traffic flow is a multi-inclusive and a complex system of interactions shaped external factors such as roadway geometry, vehicle speed, control strategies, and environmental conditions with internal factors such as driver behaviour and flow composition. Within this structure, the density of traffic stands out as the primary variable that governs travel speed, flow capacity, and overall stability of the system [1-4]. The density is a product of the traffic demand and the required time for drivers to comprehend and react to the changing conditions along the traffic flow. This article concentrates on human-operated vehicles with limited technology moving along uninterrupted lanes of roadways in one direction. Vehicles that support drivers with technology and operate with artificial intelligence (AI) will be addressed as needed in the text. This study adopts physics-based perspectives inspired by the buckling theory of columns and quantum theory. The study conceptualizes regime shifts in traffic flow through an analogy of elastic-inelastic column buckling, relating the compressive strength, stiffness, and slenderness of columns to speed, flow rates, and density of traffic flows. The study then introduces a macroscopic analogue of quantum physics concentrating on the stochastic nature of event possibilities and the quantized nature of flows.

### 2 A discussion about following distances

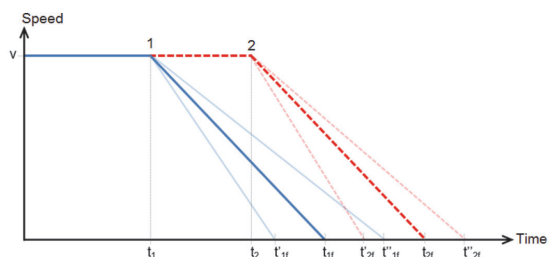
Traffic flow is composed of vehicles with varying power-to-mass ratios and vehicle dimensions. These differences result in varying acceleration and deceleration rates, varying fields of vision, and varying response times, resulting in differences in the extent of 'safe operating

zones”, referred to as the “needed awareness zones’ around a vehicle moving at the design speed of a roadway. Figure 1 represents two vehicles represented by isosceles triangles of different sizes, moving along the directions shown by the corners of their vertex angles. The differences in the sizes of the triangles depict the differences in the sizes and mass to power ratios of the vehicles. The larger vehicle that is longer, wider and higher than the smaller vehicle typically exhibits a lower power-to-mass ratio, resulting in reduced acceleration capability and different dynamic performance characteristics compared to lighter passenger vehicles [5]. A vehicle moving along a traffic lane must have peripheral control and guidance that is concentrated along the direction of its movement. This means that a clear distance must be maintained ahead of the vehicle as represented by the arrows in figure 1. These vectors,  $x(v, a, t_r, d)$ , are based on the speed of the vehicle ( $v$ ), the distance needed for the vehicle to come to a full stop from its speed that relates to vehicle deceleration ( $a$ ), the reaction time required for the vehicle operator to engage the breaks ( $t_r$ ), and the representative vehicle dimension ( $d$ ). The awareness zone surrounding each vehicle is represented by ellipses. The lateral extension of this zone is related to the field of vision, which also relates to the speed of the vehicle [6]. The backward extension of this awareness zone is shorter than the frontal extension. This elliptical representation is a simplification, as there may be an asymmetry due to the off-centered placement of the driver within the vehicle and the associated changes with the unobstructed ranges of both forward and rear views [7]. However, for the discussions presented in this study, this representation is considered sufficient.



**Figure 1** Depictions of vehicles in motion with different sizes and awareness zones that are needed surrounding the vehicles

The frontal range of this ellipse marked by  $x(v, a, t_r, d)$  provides the vehicle with the translational buffer it needs for the distance covered by the vehicle during the time that passes for the driver to react to a threat. The driver of a vehicle following another vehicle along a roadway requires a time interval to react to an imminent threat. One of these threats would be a decelerating vehicle ahead just outside the awareness zone of the following vehicle. In this case, the driver of the following vehicle must also decelerate. However, this deceleration does not occur simultaneously, but occurs after a certain time interval. Figure 2 presents velocity and time graphics of two vehicles travelling in a lane. If vehicle 1 and vehicle 2 initiate same rate decelerations at times  $t_1$  and times  $t_2$ , respectively, they stop at  $t_{1f}$  and  $t_{2f}$ . The difference between  $t_{2f}$  and  $t_{1f}$  is the headway between the two vehicles that represents the reaction time of the driver of vehicle 2 [8]. In this case, the area of the parallelogram defined by 1-2- $t_{1f}$ - $t_{2f}$  is the distance that the following car would cover within the reaction time and speed [9].



**Figure 2** Variation of velocities in time for leading (1) and following (2) vehicles

If the leading car decelerates at a higher rate such that it arrives at a full stop at time  $t'_{1f}$ , then the necessary following distance would increase to the area of the parallelogram defined by  $1-2-t'_{1f}-t_{2f}$  unless the following driver also decelerates at a higher rate and reaches a full stop at  $t'_{2f}$ . If the leading car decelerates at high rate and stops at time  $t'_{1f}$  but the following car does not decelerate at the same rate but slows down with a lower deceleration coming to a complete stop at  $t''_{2f}$ , then the following distance should accommodate the added distances due to differences in decelerations that are represented by the triangles  $1-t'_{1f}-t_{1f}$  and  $2-t_{2f}-t'_{2f}$  and increases to the polygon area represented by  $1-2-t'_{1f}-t''_{2f}$ . Based on this analytical evaluation of vehicle kinematics, the first thesis of this paper is the following:

**Thesis 1:** Decelerations of the leading and following vehicles are stochastic variables. The length accommodated for  $x(v, a, t_r, d)$  is not a deterministic parameter but is based on an assumption with respect to the decelerations of both vehicles. If the decelerations are assumed to be the same,  $x(v, a, t_r, d)$  should be the distance travelled within a given reaction time for the given running speed. If the decelerations are not the same and, for the worst case, if the leading vehicle is decelerating at a higher rate, then the following vehicle, than the following vehicle must increase its rate of deceleration as the driver experiences that the supposedly safe following distance is closing down at a rate that is unexpected. In relation to the first thesis, the second thesis is:

**Thesis 2:** The variation of the translational acceleration in time will generate a translational super-acceleration for the following driver, the value of which also can have important physiological implications that are typically not considered in ordinary traffic analysis and design. Figure 3a shows the velocity and time graphs and figure 3b shows the distance and time graphs for  $v = 90 \text{ km/h}$ , headway = 2 s and decelerations of  $a_1 = a_2 = 2 \text{ m/s}^2$ . Until the moment of breaking, which is  $t_1 = 5 \text{ s}$  for vehicle 1 and  $t_2 = 7 \text{ s}$  for vehicle 2, the distances increase linearly with time, after which they increase at decreasing rates due to decelerations. The following distance between the two vehicles is  $x = 2 \text{ s} \cdot 25 \text{ m/s} = 50 \text{ m}$ , which is also the area between the sloping parts of the two plots shown in figure 3a and also stated in figure 3b as  $x = 331.25 - 281.25 = 50 \text{ m}$ .

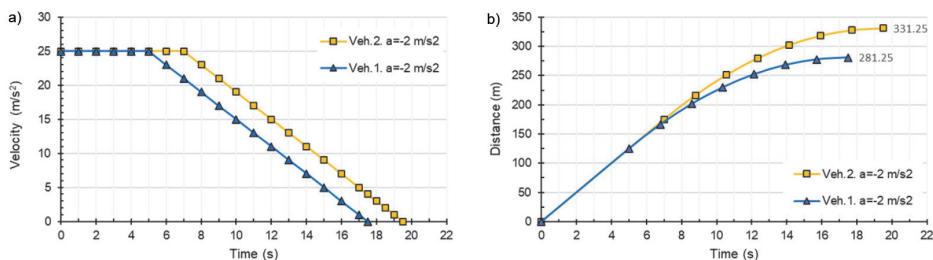


Figure 3 Variations of (a) velocities in time and (b) distances in time

A hypothetical outcome from this analysis is that if the vehicles are autonomous and interconnected such that the response reaction time of the vehicles is zero, then the headways and the associated following distances between the vehicles would no longer be needed since the following vehicles can instantly decelerate or accelerate according to the vehicle it is following.

### 3 A discussion of roadway capacities

Management of the capacity of a lane along two-lane and two-way highways and along the two-lanes of a divided highway has been an area of research since the beginning of mass production of motorized roadway vehicles [1, 10, 11]. The flow rate along a lane at a given time

interval can be determined by counting the number and type of vehicles that pass through a representative section of the lane within that time interval, which typically is a minimum 15-minutes. An hourly extrapolation based on the 15-minute count would also give the flow rate per hour. Counting through the entire hour would give the volume of vehicles served within that hour. The ratio of the hourly flow to the hourly volume would give the peak hour factor, which would indicate the fluctuation of the flow within that hour [11].

The capacity of the lane of a roadway would be the highest volume the lane can serve at a sustainable speed close to or equal to the design speed of the roadway. The variation in the behaviour of flow along an uninterrupted roadway before it reaches the capacity of the roadway and after it reaches the capacity of the roadway are two distinctly different areas of traffic flow research. The first idea is to maximize the roadway capacity and maintain the flow of the highest number of vehicles at the highest sustainable speed, which is the design speed of the roadway [11]. The second idea is to maintain flow, albeit at lower speeds, if the densities along the roadway reach levels such that the design speed cannot be sustained due to reduced following distances or safe operational intervals between the vehicles. Management of variable speed limits along roadways that reached their capacity is one measure of maintaining flow, albeit at lower speeds [12-14].

Figures 4a and 4b depict the flow along a two-lane and a one-way highway. The flow conditions are such that the longitudinal extents of the awareness zones of vehicles do not overlap and the lateral extents of the awareness zones overlap as the vehicles along opposite lanes pass each other. However, it is assumed that the lane widths and shoulder widths are sufficient to prevent lateral adversity. Each vehicle can remain in a steady flow at the design speed as long as the ellipses of each vehicle do not overlap to the extent of reducing the  $x(v, a, t_r, d)$  vectors needed for free flow. Despite the representation presented in figure 4, the quantity and distribution of vehicle sizes within a flow is not a deterministic variable.

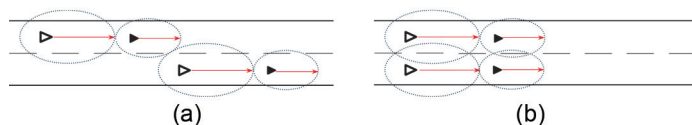


Figure 4 A simple representation of multilane traffic flow in one direction

Figure 5a shows a condition along a two lane and two-way roadway where one can state that the vehicles are independent of each other and each vehicle can reach and achieve the design speed. Figure 5b shows the condition where free-flow conditions; albeit at speeds slightly lower than the design speed, can be achieved, since the needed vector for the particular vehicle speed and dimension can be maintained. However, in figure 5c, since the ellipses overlap to the extent of reducing the spacing between the vehicles below the needed  $x(v, a, t_r, d)$ , the speeds must be reduced to accommodate the lowered  $x'(v, a, t_r, d)$ .

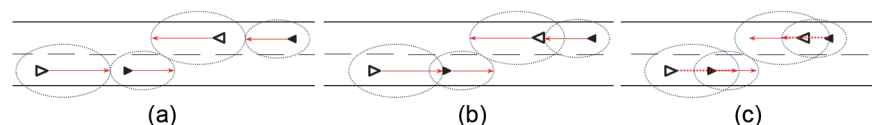


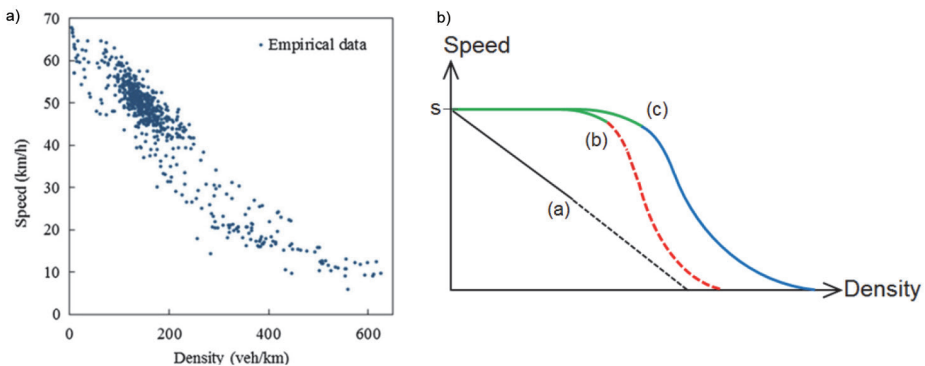
Figure 5 A simple representation of a two-way and two-lane flow

However, once the necessary awareness zones for vehicles overlap to the extent of forcing reductions in vehicle speeds, the adjustment to new lowered speeds does not occur instantly but spreads in ripples across the flow of traffic unless special measures are taken to limit the vehicle speeds. In the absence of such measures, further overlapping between individual vehicles, either by the closing of mutual gaps in between or by inclusion of a new vehicle

within an existing gap, disturb the conditions of free flow and increase the mutual awareness between each vehicle. At this stage, the movements of the vehicles are not a part of an all-encompassing free flow, but a conglomeration of moving particles with increased mutual interaction, the movements of which effect and change the movements of other exposed particles at each moment in time. As long as the flow exists for a given speed, there exists a relationship between the speed of flow ( $u$ ), the density of flow ( $k$ ) and the rate of flow ( $q$ ) as in equation (1). The parameter that sets the conditions for the flow is the density. In other words, speed and volume are outcomes of density, which is the actual presence that makes up movement known as the flow [1, 8, 10].

$$q = u \cdot k \tag{1}$$

The main idea behind the traffic management of a roadway is to accommodate the conditions to maximize the speed for the highest number of vehicles and hence maximizing the flow rate. The variation of speed with density has been and is a question for many ongoing research studies [1, 8, 10, 11]. Figure 6a presents the speed and density data obtained for a study [4]. Up to a level of density, the flow is free and governed by equation 1 and beyond that level, the flow becomes forced and finally terminates unless a variable speed management protocol is used along the roadway. Early quantification efforts to relate speed to density considered a linear variation in which design speed gradually decreased with each incremental increase in density, as shown in figure 6b [1]. Despite its simplicity and a clear representation for the variation of flow with density and flow with speed, this approach does not depict real flow conditions. Unless measures are in place to impose variable speed limits with changing roadway densities beyond a threshold, free-flow conditions transition into forced-flow conditions if the running speeds are left to individual drivers to decide. This is a stochastic process, the outcomes of which can only be evaluated statistically.



**Figure 6** a) Speed-density measurements [4], b) Representations of speed variation with density: a) Greenshields [1], b) Bell Curve [15, 16], c) Framework of the proposed CERRAHPAŞA Equation

Figure 6b presents a representation with three distinct regions, beginning with the free-flow regions, where the flow is maintained with increasing densities until a speed level that is somewhat lower than the design speed. Beyond the free flow range, the flow conditions for the design speed are no longer possible, and the flow density increases to break the flow, which eventually becomes forced before termination. When the flow density reaches a threshold beyond which the flow cannot be sustained at the design speed level, variable speed limitations can sustain the flow, although at lower speeds, as presented in figure 6b and 6c. The dashed representations for the forced-flow conditions of the previous (a) and (b) can manifest as lower-speed free-flow (c).

Let us assume a roadway lane of a multilane roadway designed for a service speed of 90 km/h, along which vehicles are free to mutually overtake and position themselves along the various lanes. Let us also assume that the flow is made up of passenger cars that are 4 m long passenger cars. A 1 km stretch of a lane would accommodate 250 passenger cars placed bumper to bumper. If all cars were controlled from a central artificial intelligence unit and if accelerations and decelerations could occur simultaneously, theoretically, the flow that a lane can accommodate would be calculated as 22,500 veh/hour (!) in equation (2).

$$q = 90 \frac{km}{h} \cdot 250 \frac{veh}{km} = 22,500 \frac{veh}{hour} \quad (2)$$

However, such a level of mutual awareness and control of vehicles is not possible when the vehicles are operated by human drivers, and the zero gaps under such conditions, allowing zero time to react, simply do not exist along human-operated flows. This being the case, equation (1), would not be valid for such low values of vehicle spacing. Human reaction time ( $t_r$ ) to an event on a roadway that would initiate deceleration is variable depending on age, health, type of vehicle operated, psychology and drivers experience [6, 7, 9]. For  $t_r = 2$  seconds, the required following distance  $x$ , assuming that the vehicle decelerations would be the same would be  $x = 2 \text{ s} \cdot 25 \text{ m/s} = 50 \text{ m}$ . This being the case, the number of vehicles ( $n$ ) that can exist within the 1 km stretch of a lane is given by equation (3) and equation (4) presents the flow that the lane can accommodate according to  $t_r = 2 \text{ s}$ .

$$(n-1) \cdot 50 \text{ m} + n \cdot 4 \text{ m} = 1000 \text{ m} \rightarrow n = 19 \text{ vehicles} \quad (3)$$

$$q = 90 \frac{km}{h} \cdot 19 \frac{veh}{km} = 1,710 \frac{veh}{hour} \quad (4)$$

If the reaction time be reduced to  $t_r = 1.6 \text{ s}$ , the necessary spacing would be reduced to  $x = 40 \text{ m}$  and the flow density of the lane would increase to  $n = 23$  vehicles and the flow rate to  $q = 2,070$  vehicles/hour. Surely, all drivers will have differences in their reaction times, and vehicles will follow at differing spacings. For any given flow case, the resultant flow rate will relate to a statistical combination of probable outcomes. Meanwhile, by using intelligent transport systems (ITS) installed along the road and within vehicles, it is possible to support the reaction times of drivers and persuade them to attain lower but sustainable speeds, and hence increase the flow density and flow rate. The following two sections introduce two approaches to traffic flow inquiry from the realms of classical mechanics and quantum mechanics.

## 4 Classical mechanics analogies: plastic and euler buckling

An event observed within a scientific discipline can benefit from the understanding and support that another discipline can provide. Variation of speed with density and the transition of free flow into forced flow and the ITS support to enhance and sustain the conditions for free-flow has a strong analogy in compressive strength assessment of columns in relation to column slenderness. Figure 7a presents the variation of the critical axial compressive force ( $P_{cr}$ ) that a column with the slenderness of the column for a given yield strength [17]. Figure 7b presents the variation of the buckling curves for different strength column materials. The variation presented in figure 7a has three distinct regions: 1. post-yield behaviour, 2. post-yield buckling, 3. Euler (elastic) buckling [12]. The elastic compressive strength of a column with very low slenderness is limited by the cross-sectional area of the column and the elastic yield stress limit of the column material. A column with a very low slenderness ratio would not buckle and the structural cross section can withstand the imposed stresses up to and beyond its yield limit ( $\sigma_y$ ).

The critical compressive force within this region is equal to or greater than the compressive yield force ( $P_y$ ) of the column. However, beyond a certain slenderness, parts of the column cross section cannot fully reach the yield limit because it buckles out of plane. As the slenderness increases, the compressive force below the yielding force that a column can carry before it elastically buckles is given by the Euler equation presented in equation (5) [17, 18]. In this equation,  $L$  is the length of the column,  $r$  is the radius of gyration of the cross section along the buckling axis,  $K$  is the effective length factor due to the boundary conditions, and  $E$  is the elastic modulus of the column material. Part of the possible effective length factors is included in figure 7b for convenience [17]. The slenderness of a given length and cross section can be improved by the support conditions at the ends of a column or intermediate supports along the length of the column. Therefore,  $K$  is a powerful parameter that can enhance the slenderness if applied properly but can also reduce the slenderness if applied improperly. Equation (6) gives the compressive elastic force according to Euler's buckling stresses where  $A_g$  is the cross-sectional area of the column. Equation (7) is a qualitative representation of the Euler equation that relates the structural aspects of a column under compression. Overlain in figure 7a is a proposed equation  $R = f(C, D, M)$  that will be explained.

$$\sigma_e = \frac{\pi^2 E}{\left(K \frac{L}{r}\right)^2} \tag{5}$$

$$P_e = \sigma_e \cdot A_g \tag{6}$$

$$\text{Strength} \propto \frac{\text{Stiffness}}{\text{Slenderness}} \tag{7}$$

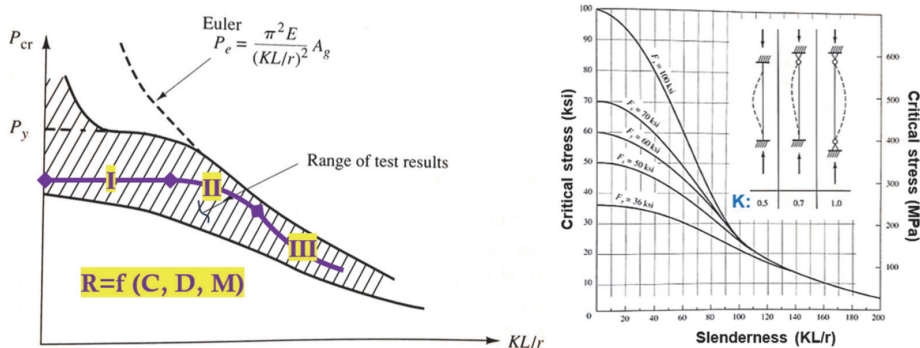


Figure 7 a) Variation of the compressive strength with slenderness [17], b) variation of the buckling curves for different strength of the column steel [17]

The Euler equation results in an infinite axial compressive strength as the slenderness of the column ( $L/r$ ) approaches zero as  $L$  approaches zero. Since there is a physical limit to this mathematical equation, the maximum force the column can reach is related to  $A_g$  and the yield strength of the column material  $\sigma_y$  as shown in equation (8).

$$P_y = \sigma_y \cdot A_g \tag{8}$$

If the material is forced into the strain hardening region beyond the yield point, the amount of compressive force that can be generated can also increase beyond  $P_y$ . However, for simplicity, the force that causes the yield will be accepted as the highest compressive force that the column can sustain. Between the region defined by equation (8) and the region defined by equations (5) and (7), there lies an intermediate region where buckling occurs inelastically. The comparison of figure 7a with figure 6 and the comparison of equation (7) with equation 1 provide a new insight. The strength of the column for a given slenderness is the highest sustainable stress that can be correlated to the speed that can be sustained along a roadway with flow conditions. The stiffness of the column is correlated with the highest speed for the highest sustainable flow rate that the roadway can permit and the slenderness of the column is correlated with the density along a roadway that can be managed. This correlation is stated in equation (9).

$$\sigma_e = \frac{\pi^2 E}{\left(K \frac{L}{r}\right)^2} \rightarrow \text{STRENGTH} = \frac{\text{Stiffness}}{\text{Slenderness}} \cdot s = \frac{v}{d} \rightarrow \text{SPEED} = \frac{\text{Flowrate}}{\text{Density}} \quad (9)$$

The third thesis of this paper based on this correlation is as follows:

**Thesis 3:** “From a mechanistic point of view, the strength ( $\sigma$ ) can represent the speed of vehicles ( $u$ ), the stiffness ( $E$ ) can represent the characteristic of the road stated in terms of the highest achievable speed along the roadway ( $C$ ) and the slenderness ( $L/r$ ) can represent the characteristic of density along the lane that in return is related to demand ( $D$ ). The effective length factor ( $K$ ) can represent the means and measures of management ( $M$ ) such as intelligent transport systems that include variable speed limits and early warning systems that can improve flow conditions to achieve higher densities along a given lane despite lower speeds.” Figure 7b has a very important implication that the flow conditions along the roadway lanes designed to support very high speeds (increasing critical stress analogy for columns) have a lower speed sustainability for increasing densities. Therefore, the fourth thesis of this paper is the following:

**Thesis 4:** Roads designed for lower speeds can maintain that speed for a higher vehicle density as opposed to roadways designed for higher speeds. This may be related to the difficulty in maintaining the necessary headway within the flow at higher speeds. In other words, all else being the same, along a kilometer or a mile long roadway lane, higher design speeds will permit lower densities.

The Reserve = Function of (Capacity, Demand, Management) equation embedded in figure 7a, presents the conceptual framework for the “Cerrahpaşa Equation:  $R = F(C, D, M)$ ” that is currently being developed. The highway speed reserve ( $R$ ) is represented as a function of built-in roadway capacity for the highest possible speed  $C(\alpha, u_d)$  based on geometry and human physiology, the demand for the roadway  $D(k, k_m)$  that reflects the density and the traffic management infrastructure installed along the roadway and  $M(\beta)$  that can improve the built-in capacity. Based on this mechanical analogy, the suggested relationship can be structuralized through equation (10), which provides a closed-form expression for the operating speed as a direct function of traffic density where  $u$  denotes the running speed of vehicles,  $u_d$  represents the design speed determined by the geometry and human constraints,  $k$  is the traffic density,  $k_m$  reflects the ‘manageable density’ that marks the transition from free-flow to forced-flow conditions and “ $\alpha$ ”, “ $\beta$ ” and “ $n$ ” are numeric values reserved for particularities of the analysed roadway in terms of capacity, constraints, and managerial aspects.

$$u = \frac{\alpha \cdot u_d}{1 + \left( \beta \frac{k}{k_m} \right)^n} \quad (10)$$

The Cerrahpaşa Equation; currently under development, has the potential to transform the qualitative mechanical analogy presented above into a compact analytical model that naturally produces three flow regimes while preserving mathematical simplicity comparable to classical fundamental diagram models.

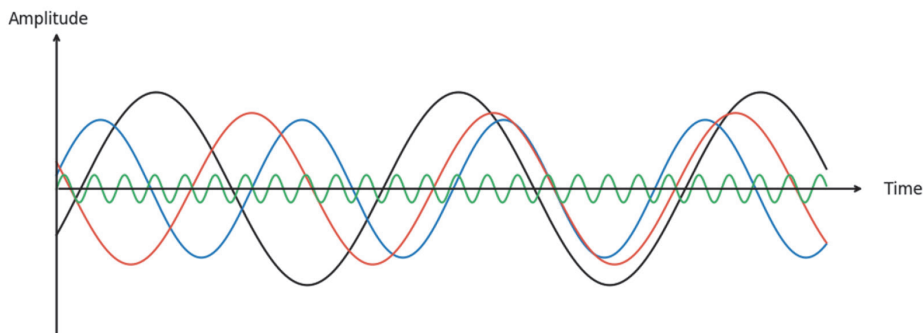
## 5 Quantum mechanics analogies: Quants and Continuity

The realm of quantum mechanics brings us closer to a better understanding of the nature of our physical presence. When an understanding based on absoluteness of time, determinacy of events, and continuity of nature is challenged with the behaviors observed in the physical world at high speeds, high frequencies, and infinitesimal masses, a relative, stochastic, and quantized explanation of nature rises to the challenge. In the late 19<sup>th</sup> century, observations of physical world events, such as black body radiation and photoelectricity, could not be explained by an understanding based only on the continuous emission of electromagnetic radiation [19, 20]. A new understanding based on quantized nature of energy and its possible states of being led to the development of the realm of quantum physics that has enhanced our perception and understanding of the physical world. The comprehension of the quantized nature of energy and particle-wave duality of matter is challenging. Yet, the reality of our physical world that we experience deterministically for our purposes on the macro-scale bases on a delicate web of intertwined micro principles many of which are yet to be discovered and some of which can only be stated within a room of uncertainty.

Quantum mechanics suggests a wealth of new findings that await comprehension. One such finding is that means of observation affect what one observes. Hence, a picture of a reality requires many different forms of observation. Another is that one may speak about the many possible states of a being rather than certainties of the states of a being. In essence, despite the temptation to borrow or search for reflections of such explanations in our macro world, it can be a futile attempt.

A probabilistic representation of the states of matter and energy has easier prospects of finding analogies in the world of classical mechanics, where determinacy is also questionable and probabilities exist. However, despite the difficulties in correlating the totality of quantum physics to the world of classical mechanics, new explanations and characterizations that quantum physics offers to explain energy and matter, when taken individually, can support the development of new understandings for the realm of classical mechanics [21]. One may address the physical realities of a moving vehicle in addition to its most obvious mechanical fact that it is a moving mass with direction and speed. This embodiment of mass and kinetic energy can be portrayed with the frequency and wavelengths of the sounds it generates. Taking a cue from the de Broglie hypothesis [22], if a person were to stand near a highway lane with free-flowing traffic under a certain climate condition (since air density of air affects the speed of sound), within an open space (since enclosed spaces, such as tunnels can change the qualities of sound waves) and record the sounds of the passing vehicles, he or she would hear a colloquium of gradually increasing and decreasing intensities of sound waves. The wavelength of the sound waves would relate to the density and speed of the flow as well as the stochastic variations in headways between the approaching vehicles along with geometry of the vehicles, size and aerodynamics of the vehicles, and whether the vehicles are powered by fossil fuel engines or electric batteries (and perhaps in the future hydrogen cells).

Figure 8 presents a representative set of sinusoidal sound waves with varying wavelengths and varying amplitudes. Within this sound signal, there also exists a higher frequency content in relation to the vehicle engine, the wavelength of which is independent of the headway but the intensity of which relates to the headway. In other words, the energy and momentum of a moving vehicle also have the other quality of accompanying waves of varying frequency and wavelength.



**Figure 8** Representation of the assumed sound wave profile for free-flowing traffic

However, one should be careful when borrowing terms and qualities from the realm of quantum mechanics, since in this case, it is not the moving vehicle that is acting as a mass and a wave at the same time as in quantum understanding, but only the moving mass is generating sound waves that are two separate qualities and which are strictly within the realm of classical mechanics. Along a roadway lane with forced flowing traffic, the same observer will likely report a different variation in the sound ways he or she is receiving, as illustrated in figure 9. In this case, the frequency content of the free-flowing vehicles will not be present. The observer will receive the high frequency content of the sounds generated by the engines that rapidly increase and decrease as they decelerate, stop and accelerate as part of the forced flow.



**Figure 9** Sound wave profile representation for forced flow traffic

In essence, while the sound produced by the engines is always present, their relative effect within the flow conditions changes. Where one can state that for the free-flowing traffic, the character of the sound is mostly harmonically variable, this character transforms into one with transforms into one with characteristic of controlled-flow conditions, as shown in figure 10.

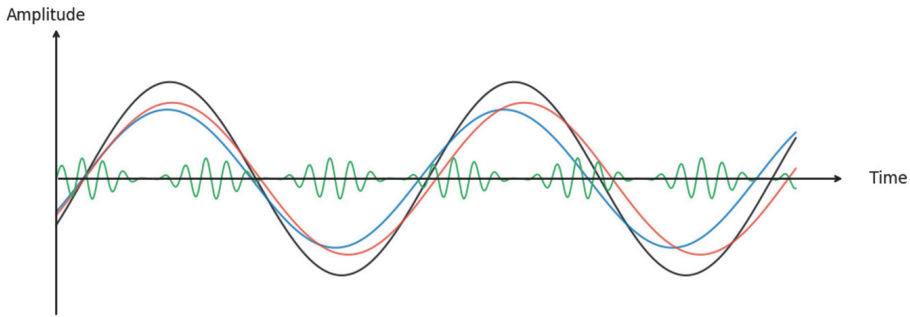


Figure 10 Representation of assumed sound wave profile for controlled-flow

Therefore, the fifth thesis of this paper is as follows:

**Thesis 5:** It is expected that the colloquium sound waves generated by traffic flow have certain characteristics with regard to their frequency, amplitude, and wavelength content. These characteristics change with the flow conditions as they vary from free-flow to forced-flow, unless traffic and wheel management systems are in place to prevent forced flow and sustain flow at speeds lower than the design speed. Such a register of the emanated sound waves can provide the means to measure and control the traffic flow.

Schrödinger's Equation; the development is rooted in de Broglie hypothesis, provides us with an understanding of how to represent the condition of a system in time and domain with a "wave function  $\Psi(x, t)$ " [22, 23]. In the meantime, one can investigate the presence of a vehicle within a traffic flow in relation to the speed and density of the flow. Headway, which is an important parameter that affects the flow conditions, is a boundary in time that affects the conditions of flow if trespassed. If an intrusion occurs into the time distance represented by the headway, the flow conditions are adversely affected. In other words, the intruded vehicle no longer acts as a part of the flow (wave) but acts as an individual entity (mass) within the disturbed flow.

## 6 Conclusion

This paper revisited the conditions of flow in traffic engineering with new insights borrowed from the realms of classical mechanics and quantum mechanics. A new insight into the reaction times is developed and five separate theses are introduced about the density-speed-flow characteristics of traffic. The discussions and suggestions shared in this study form the basis for future studies. The concept of reaction times is revisited. Based on the analytical studies presented, the authors propose that the reaction time should be studied in two phases, namely: 1. Time needed to react (decelerate), 2. Time required to establish proper deceleration. Nowadays, advanced sensor technology installed in vehicles assists the driver with the correct deceleration, but in their absence, roadway designers and roadway managers must address this clear kinematic condition.

A new insight into traffic flow theory is suggested through the physics of column buckling. Another new insight is introduced through the sound wave content of the traffic. The end of this study is the beginning of many other distinct studies based on introduced perspectives on physics.

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