

AN ALTERNATIVE ANALYSIS FOR DEVELOPING THE SWELLING MODEL FOR EXPANSIVE CLAYS

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Abstract

Pavements and railway beds are badly affected by the behavior of their expansive clay subgrades. Seasonal drying and wetting are particularly responsible for irregularities in the pavement surfaces through differential settlement and heaving. In Israel, a method of quantifying the amount of heave expected is based on one-dimensional laboratory swelling curves. Since time, site, and budget limitations frequently do not allow complete laboratory testing, empirical correlations are commonly used instead of the one-dimensional laboratory curves. In local studies conducted in 1969 and 1985, these correlations yielded the required general swelling model for any given clay characteristics. This general swelling model has recently been updated by applying the Excel–solver analysis to new local test results from, all together, 897 undisturbed specimens. The present paper, in addition to further updating the model, describes an alternative analysis carried out on these local test results. This alternative analysis is based on the following two-stage operation: (a) conducting a multiple linear regression on the swelling–pressure tests results (i.e., the ASTM 4546 Method c test results) to obtain the swelling–pressure correlation for any given clay characteristics; (b) utilizing the correlative equation obtained in stage a to perform an additional linear regression on the swelling–percentage test results (i.e., the ASTM 4546 Method B test results) with a single independent variable, defined by the given surcharge pressure divided by the predicted value of the swelling–pressure.. Finally a comparison of these two general swelling models indicates a preference for the existing model, generated from the Excel–solver analysis.

*Keywords: Excel–solver, expansive clay, heave, swelling model,
swelling percentage, swelling pressure*

1 Introduction

Expansive soils are a worldwide problem that poses several challenges for civil engineers. Pavements and railway beds constructed on these clays are subjected to large uplift forces caused by swelling generated from moisture variation. These uplift forces induce heaving and cracklings to the surface of these structures as shown in Figure 1.

Researchers and engineers have for several years been occupied with the engineering problems presented by swelling clays. The research and the experience that developed, which have found expression in many technical publications, in effect highlight basic differences among the approaches practiced in various countries, whether Australia, India, Israel, Nigeria, South Africa, USA (Texas, Kansas, etc.), Turkey, and others, in particular with regard to clay as a subgrade material for flexible pavements. Consequently, these various approaches should be considered and adjusted to local needs.



Figure 1 Severe longitudinal cracking and differential settlement and heaving taking place on the surface of a pavement based on an expansive clayey subgrade

In Israel, theoretical and practical engineering experience has accumulated on the construction of such pavements since 1956. The initial basic research, together with the knowledge of the performance of numerous pavements that accrued, led to the crystallization and adoption of engineering solutions for the construction of pavements on these soils; see, for example, [1–5]. In other words, these activities yielded a local procedure for designing flexible pavements on swelling clays. This procedure has been suggested for inclusion in the Israeli PWD guide for the design of flexible–pavement structures.

Predicting the heave of a pavement surface as a result of subgrade swelling constitutes a basic feature of the Israeli procedure. The prediction necessitates knowledge of the swelling–pressure characteristic (curves) of the clay strata under consideration, which is usually obtained from laboratory tests on undisturbed clay specimens. These swelling curves are dependent on knowledge of the following parameters that characterize the clay being studied: (a) its liquid limit, (b) the ratio between its in–situ moisture content and its plasticity limit, and (c) its in–situ dry density.

Previous discussions indicate that the development of the local swelling model is very important for calculating both heave and adequate surcharge pressure. The latest local swelling model was developed in 2011 [6 and 7] on the basis of 352 local test–results. Since then, 545 new test results have become available, enabling an updating of the swelling model. In light of all the above, the objectives of this paper are as follows:

- updating the swelling model published in 2011 [6 and 7] to predict vertical swell under a given vertical pressure exerted on the clay under consideration;
- developing an alternative swelling model following the local computational procedures published in 1985 [3];
- comparing the two adjusted swelling models and selecting one for the final routine calculations.

The sections to follow will detail the process of attaining these three objectives and their associated conclusions. Finally, the following quotation from the song by George and Ira Gershwin seems appropriate here: '...In time the Rockies may crumble, Gibraltar may tumble, there're only made of clay, but our love is here to stay....'

2 Review of the existing model

In [2 and 3], 352 undisturbed samples were used to determine the direct dependence of vertical swelling, in percentages, with the following variables that characterize undisturbed clay samples: liquid limit (LL) in percentage, moisture content (w) in percentage, dry density in kN/m³, and finally the applied vertical pressure (Pp) in kPa. This determination was performed with the Excel–Solver command, utilizing (a) the first Israeli general swelling–pressure model arrived at in [2], since the linear multiple regression was conducted on 125 undisturbed samples in 1969; and (b) the basic vertical swelling model arrived at in [3] after a series of empirical relationships reported in the technical literature was analyzed in 1985 but no requested verification was received from local laboratory tests.

In the studies of [6 and 7], two replacements were utilized in order to obtain a higher coefficient of determination (R²): log(LL) instead of LL and W/PL instead of w. This led to the two following equations:

$$\log(P_o / 98.07) = -3.256 + 1.540 \times \log(LL) - 0.537 \times W / PL + 0.738 \times (D / 9.81) \quad (1)$$

$$S_p = -1.872 \times (P_o / 98.07) \times \log(P_p / P_o) \quad (2)$$

The standard error (SE) value associated with the development of these two equations is 1.45%, and the coefficient of determination (R²) obtained for them is rather low, 0.387. This value, however, is at the same order of magnitude as that obtained for the multiple regression analysis performed on 514 undisturbed samples in Kansas (i.e., 0.35 [8]). Here it should be noted that the advantage of eqn (2) over the original equation of [3] is that in contrast to the original equation, eqn (2) is based on a statistical analysis of the local test–results.

The significance of the SE value of 1.45% is that 68% of the total vertical swelling predictions from eqns (1) and (2) are expected to be accurate within a range of ±1.45% of the values calculated. Similarly, 95% of these total predictions are expected to be accurate within a range of ±2.90% of the calculated values. Both these ranges indicate the existence of an extensive dispersion characteristic in the measurement results.

3 Development of the updated models

The previous 352 test results and the new 545 test results (i.e., 897 test results in all) became available for updating the swelling model with the Excel–Solver command. This updating led to the following two equations:

$$\log(P_o / 98.07) = -4.234 + 2.110 \times \log(LL) - 0.399 \times W / PL + 0.604 \times (D / 9.81) \quad (3)$$

$$S_p = -2.191 \times (P_o / 98.07) \times \log(P_p / P_o) \quad (4)$$

The standard error (SE) value associated with the development of these two equations is 1.952%, and the coefficient of determination (R²) obtained for them is higher than that (0.604) associated with eqns (1) and (2).

Figure 2 summarizes the overall prediction accuracy of the swelling model given by eqns (3) and (4). A measure of overall bias in this model relates to how closely the unconstrained linear regression line of predicted versus measured vertical swelling matches the line of equality; i.e., how close are the unconstrained intercept and the slope to 0 and 1, respectively.

Thus, Figure 2 indicates that the unconstrained regression line has an almost considerable intercept (S_p=0.787%) and a slope lower than the value of 1 (0.596), thus, exhibiting an almost considerable bias and, on the average, poor similarity between measured and predicted values.

In addition to the above updating of the model, an alternative analysis was carried out on the 897 local test results. This alternative analysis is based on the following double-stage operation: (a) conducting a multiple linear regression on the swelling–pressure test results (i.e., the ASTM 4546 Method C test results) to obtain the swelling–pressure correlation for any of the given clay characteristics listed above; (b) performing an additional linear regression on the swelling–percentage test results (i.e., the ASTM 4546 Method B test results) with a single independent variable, defined by the given surcharge pressure divided by the predicted value of the swelling–pressure and utilizing the correlative equation obtained in the previous stage. The first stage, described above, was carried out on 362 of the total 897 undisturbed samples. Obviously for these undisturbed samples, the vertical swelling was kept at zero. This led to the swelling–pressure model given by the following equation:

$$\log(P_o/98.07) = -6.382 + 2.619 \times \log(LL) - 0.226 \times W/PL + 1.161 \times (D/9.81) \quad (5)$$

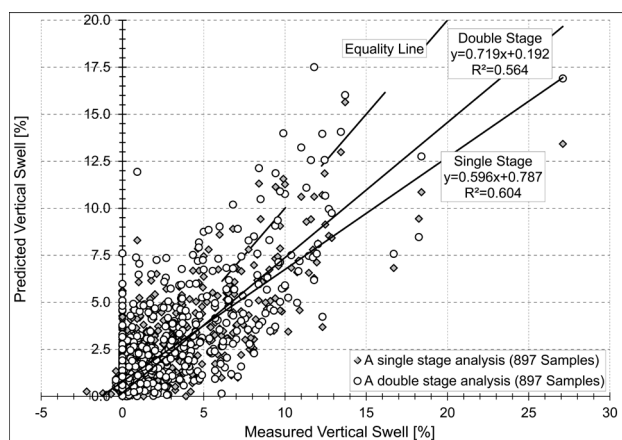


Figure 2 Predicted versus measured vertical swelling of 897 undisturbed samples for the single-stage analysis (i.e., eqns 3 and 4) and the double-stage analysis (i.e., eqns 5 and 6)

The standard error (SE) value associated with the development of this equation is 0.394, and the coefficient of determination (R^2) obtained for it is rather low, 0.339. The significance of the 0.394 value for SE is that 68% of total P_o predictions from eqn (5) are expected to be accurate within a range of $10^{-0.394}$ (i.e., 0.458) to $10^{0.394}$ (i.e., 2.182) times the value calculated. Similarly, 95% of these total P_o predictions are expected to be accurate within 0.210 up to 4.763 times the calculated value. Both these ranges indicate again the existence of an extensive dispersion characteristic in the measurement results, as do the low value values obtained for R^2 . The execution of the second stage is defined by conducting a zero–intercept linear regression on the available vertical swelling test results (i.e., the ASTM 4546 Method B test results) of the remaining 535 undisturbed samples. This leads to the vertical swelling model given by the following equation:

$$Sp = -4.429 \times (P_o/98.07) \times \log(P_p/P_o) \quad (6)$$

This zero–intercept linear regression is shown graphically in Figure 3. This figure indicates that the dispersion of the results is considerable, leading to a low coefficient of determination (R^2) of 0.544.

Finally, the prediction of vertical swelling from the combination of eqns (5) and (6) results in a standard error (SE) value of 2.171%, which is higher than that associated with eqns (3) and (4), and a coefficient of determination (R^2) value of 0.510, which is lower than that associated

again with eqns (3) and (4). This in itself leads to the conclusion that the application of eqns (3) and (4) for predicting vertical swelling values is preferable.

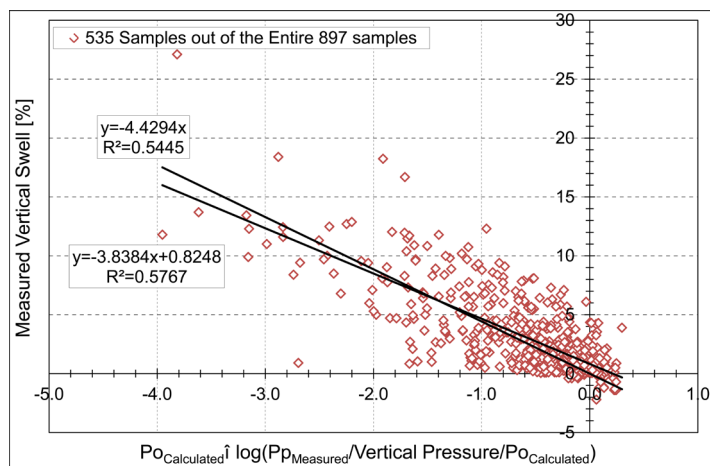


Figure 3 Measured vertical swelling versus $P_o \times \log(P_p/P_o)$ of 535 undisturbed samples, where P_o is the calculated swelling–pressure according to eqn (5) and P_p is the measured (applied) vertical pressure

Figure 2 also summarizes the overall prediction accuracy of the swelling model given by eqns (5) and (6). The figure indicates that the unconstrained regression line has practically a zero intercept ($S_p=0.192\%$) and a slope value of less than 1 (0.719), thus exhibiting zero bias and, on the average, poor similarity, again, between measured predicted values. These findings are somewhat preferable to those associated with the single stage analysis, although the R^2 value of this analysis (0.604) is higher than that of the double stage analysis (0.564)

Finally, the comparison of the predicted outputs from the three swelling models given is discussed in the following section, which also describes the associated practical conclusions.

4 Comparisons and conclusions

Figure 4 shows a graphical comparison of vertical swelling versus vertical pressure for (swelling models) calculated with the aid of (a) eqns (1) and (2) as derived from 352 undisturbed samples; (b) eqns (3) and (4) as derived from 897 undisturbed samples; and (c) eqns (5) and (6) as derived again from 897 undisturbed samples. The last calculation is done according to the 2–stage procedure; and all three are calculated for a case in which the dry density is equal to 14.7 kN/m^3 and the ratio of moisture content to plasticity limit equals 0.8.

The figure indicates that the transition from 352 to 897 undisturbed samples does not radically change the pattern of the vertical swelling variation. This indication may suggest that the two basic, predefined equations given in [2] and [3], together with the regression procedure (i.e., the application of the Excel–solver analysis), are appropriate.

In addition, Figure 4 shows that the transition from a single–stage analysis to a double–stage analysis of the same 897 undisturbed samples dramatically changes the pattern of the vertical swelling variation. In particular, it considerably reduces the swelling–pressure values. This change may suggest that the application of a double–stage analysis is questionable.

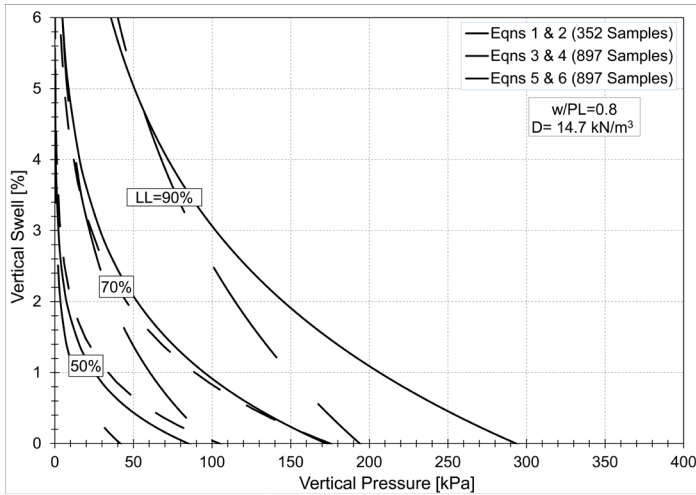


Figure 4 Comparison of vertical swelling versus vertical pressure equations (swelling models) for a dry density of 14.7 kN/m³ and w/PL=0.8

The summary–statistics of all developments described in the preceding sections is shown in Table 1. To recall, the data in this table are given for the two independent variables, log(LL) and W/PL (i.e., instead of the original LL and w). When the principle of a minimum standard error (SE) is applied to the 897 undisturbed samples, the preferred solution is given by eqns (3) and (4).

Table 1 Summary of the obtained SE, SE/SY, and R² values

Section No.	Eqns No.	Number of Samples	Analysis Method	SE %	SE/SY	R ²
2	1 & 2	352	Excel–Solver	1.45	0.788	0.387
3	3 & 4	897	Excel–Solver	1.95	0.631	0.604
3	5 & 6	897	Double Stage	2.17	0.702	0.510

It is interesting to note that Table 1 indicates that the Excel–Solver solution for the 352 undisturbed samples leads to a lower value of SE than does the Excel–Solver solution for the 897 undisturbed samples although the associated R² value decreases. This lower value, however, is not surprising, because the SE/SY value for the 352 undisturbed samples is higher than that for the 897 undisturbed samples. Note: the SY value denotes the standard deviation of measured Sp values. Another conclusion that derives from both Table 1 and Figure 4 is that for the same number of samples, even a small deviation from the minimum SE principle (i.e., 2.17% versus the minimum 1.95%) leads to considerable changes in both vertical swelling and swelling–pressure patterns.

Here, it is interesting to compare the goodness–of–fit statistics of Table 1 with those given in Table 2. This comparison leads to the conclusion that the single–stage analysis and the double–stage analysis (both on 897 samples) can each be categorized as a fair correlation analysis; however, the single–stage analysis is nearer to the good correlation criterion than is the double–stage analysis.

Table 2 Criterion for correlation as rated by goodness-of-fit statistics (SE/SY and R2) taken from [9]

Criterion for Correlation	Excellent	Good	Fair	Poor	Very Poor
R ²	≥0.90	0.70–0.89	0.40–0.69	0.20–0.39	≤0.19
SE/SY	≤0.35	0.36–0.55	0.56–0.75	0.76–0.90	≥0.91

Finally, the findings of this paper make it clear that the single-stage operation of the Excel-Solver analysis is preferable to the double-stage operation, consisting as it does of (a) a multiple linear regression analysis for measured swelling–pressure values and (b) a constrained zero–intercept linear regression analysis for measured vertical swelling values (all swelling values except those for zero–value).

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