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# Road and Rail Infrastructure V

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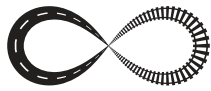
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## ELASTIC WAVES IN THE RAILROAD TRACK SUBSTRUCTURES AND ITS SURROUNDINGS ANALYZED WITH NON-CLASSICAL OPERATIONAL METHODS

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### Abstract

We analyze the propagation of the waves generated by the rolling stock and the interaction of those waves on the medium and its surroundings. We use non-classical operational methods for monitoring construction of the railway infrastructure and for noise damping.

*Keywords: elastic waves, railway substructure, non-classical operational methods, vibrations, noise*

### 1 Introduction

A railway track operates in a specific physical environment including the soil and the air. From the mechanical point of view, those media can be regarded as Infinite elastic media. What is more, the materials that the track, railroad bed and railway vehicles are made of are also elastic. The interaction of between the track and the vehicle (wheel – track) causes vibrations of air particles as well as the entire system. The vibrations are transferred to the railroad bed and further to the subsoil [1].

It is important to point out that the vibrations in the soil- and air-like medium generated by the passing rail vehicle can occur even when the vehicle already left the selected section of railway line. However, those vibrations cannot be present infinitely long due to the dissipative forces. The elastic waves propagating due to the rolling stock can be measured using laser vibrometry [2].

We are interested in the waves generated by the passing trains. Information about the loads propagate in the railroad bed and the subsoil through the waves that are actually the perturbations of velocity fields. In the case of static or quasi-static loads, we will not consider the impact of waves since the load period is much longer than the wave emission time. Elastic waves are the mechanical waves propagating in the elastic medium such as railroad bed, and they are transferred to the surroundings due to the action of the forces associated with the volumetric and / or shape deformation of the elements of the entire medium. The trains are the external objects that cause the deformations, which act as the sources of the waves. Track geometry also influence the wave generation [3-5].

The propagation of elastic waves is based on the excitation of the medium particles located away from the source of the waves. The main characteristic that allows to recognize the elastic waves from any other ordered motion of medium particles is the fact that propagation of elastic waves of low amplitudes is not associated with any transport of the substance. Moreover, the elastic waves with short duration and large amplitude are described as shock waves [6]. The elastic waves can be classified as bulk waves propagating in the soil and in the air (noise) and the surface waves propagating at the interface between the media of different properties (e.g. waves propagating at the surface of the soil (Fig. 1).

Longitudinal bulk P-waves are the first to reach the vibration analyzer. In this case, the direction of wave propagation is parallel to the motion of medium particles causing compression and rarefaction of the medium. Let us indicate the velocity of P-waves as  $c_L$ . Transverse bulk S-waves reach the detector after the P-waves, and their velocity will be given by  $c_S$ . The velocities of P- and S-waves depend on the parameters of elasticity of the medium, and the difference between the speeds can be significant (it can be linked to the depth in the soil). The bulk waves can be reflected or refracted when they pass between two media of different elastic properties. The refraction and reflection of the waves is associated with generation of both types of waves (Fig. 2).

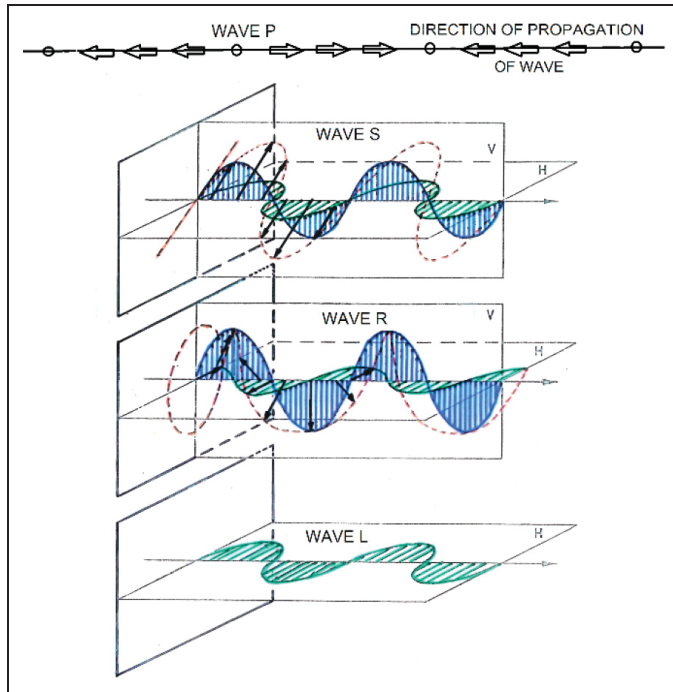


Figure 1 Waves

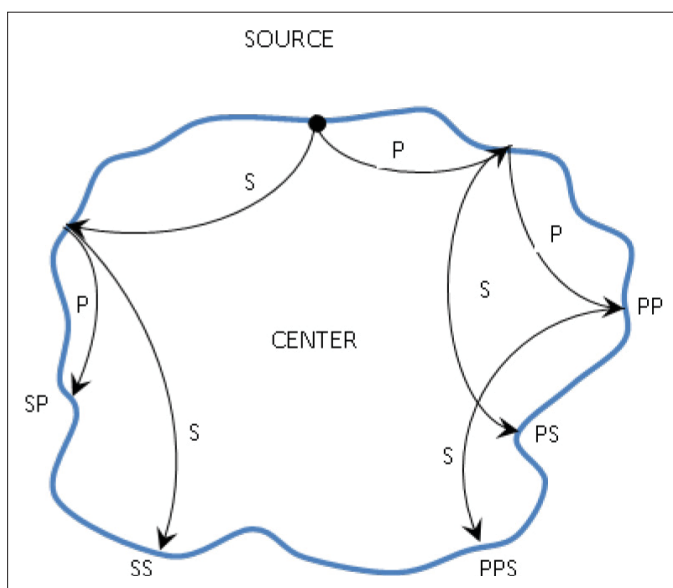


Figure 2 Propagation of bulk waves in the medium. PS – waves reflected at the interface of the medium (incident P, reflected S); SP – waves reflected at the interface of the medium (incident S, reflected P); PP – waves reflected at the interface of the medium (incident P, reflected P); SS – waves reflected at the interface of the medium (incident S, reflected S); PPS – waves after double reflection.

Rayleigh and Love surface waves (R- and L-waves, respectively) have usually long periods and variable amplitudes. However, vibration amplitude of R and L waves decreases exponentially with the depth. Rayleigh waves propagate in the horizontal direction and cause both vertical and horizontal (but not really transverse) movements of the soil surface (Fig. 1). However, the vertical and horizontal components are out-of-phase so that the movement of the particle is elliptical. In other words, the movement of an elastic particle creates an ellipse oriented vertically, which is perpendicular to the direction of wave propagation. Love waves L also propagate in horizontal direction but they cause transverse, horizontal movements of the particles (Fig. 1). The elastic waves can be generated intentionally or unintentionally. The first group of waves with characteristic parameters are used to monitor the construction of buildings, bridges, pipes, rail systems and vehicles or their elements [7]. The second group of waves are usually undesired side effects of other actions (processes) and have negative impact on the human being and the environment. The waves generated unintentionally are measured or used in a passive experiment to detect them, to eliminate them (vibration eliminators) as well as to alleviate the effects of environmental impact. They are also used to monitor the technical state of the civil engineering constructions during their operations (e.g. when the train is running, air planes etc.).

## 2 Wave equation

The waves are observed as the deviations of particles (of the medium) from their equilibrium due to the vibrations. The wave motion is actually a transfer of mechanic energy that can be described using partial differential equations. Let us consider the following equilibrium equation to derive one-dimensional wave equation [8]:

$$\frac{\partial \sigma_x}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (1)$$

Using linear Hooke law  $\sigma_x = E \varepsilon_x$  and assuming linear relations in the geometric equation

$\varepsilon_x = \frac{\partial u}{\partial x}$  we can obtain one-dimensional equation of elastic wave:

$$\frac{\partial}{\partial x} E \left( \frac{\partial u}{\partial x} \right) = \rho \frac{\partial^2 u}{\partial t^2} \quad (2)$$

If we assume that E does not depend on x, one-dimensional elastic wave equation can be written in the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E} \frac{\partial^2 u}{\partial t^2} \quad (3)$$

If we substitute the following relation for the velocity of longitudinal P-wave to Eq. (5):

$$c_L = \sqrt{\frac{E}{\rho}} \quad (4)$$

we can obtain differentia equation describing one-dimensional longitudinal elastic wave:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_L^2} \frac{\partial^2 u}{\partial t^2} \quad \text{or} \quad \frac{\partial^2 u}{\partial t^2} - c_L^2 \frac{\partial^2 u}{\partial x^2} = 0 \quad (5)$$

When we want to include a source in wave equation, then it is necessary to add a function  $f(x,t)$  to Eqs. (2), (3), (5). The function  $f(x,t)$  is called a source component. This leads to the inhomogeneous partial differential equation [9], [10]. Generally, the wave equation has the form:

$$Wu = f(x,y,z,t) \quad (6)$$

where: 
$$W = V - \frac{1}{a^2} \frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{a^2} \frac{\partial^2}{\partial t^2}; a \neq 0$$

since: 
$$V = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Attention! In the case of  $n=2$ , Laplace operator is reduced to  $V = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  and for  $n=1$  we obtain the operation  $\frac{\partial^2}{\partial x^2}$ , which changes differential equation (6) of the model into e.g. the

equation of the type (5) with the source function. The variables  $x, y, z$  are the coordinates of the point  $P \in \Omega$  at the time  $t$ , and  $u$  is a function of the appropriate class in the set  $\Omega \times \langle 0, \infty \rangle$  and indicates the displacement (change of the location of the points  $P$ ). Solving this type of equations is complex and requires numerical approaches, including finite element method [11]. Non-classical operator calculus can be used to solve the wave equations [12], [13], [14], especially with the generalized Taylor formula:

$$u = su + TsSu + T^2sS^2u + \dots + T^{n-1}sS^{n-1}u + T^nsS^nu, \quad (7)$$

$$u \in L^n = \{u \in L^{n-1} : Su \in L^{n-1}\}$$

in which  $S, s$  and  $T$  are three arbitrary linear operations so that  $S: L^1 \rightarrow L^0, T: L^0 \rightarrow L^1, s: L^1 \rightarrow \text{Ker}S$  defined on two linear spaces  $L^1, L^0$ , where  $L^1 \subset L^0$ . Moreover, the operations  $S, s, T$  must fulfil the conditions  $ST = \text{id}, TS = \text{id} - s$ . They also include the operations given by:

$$S = \frac{\partial^2}{\partial t^2} - a^2 \frac{\partial^2}{\partial x^2} \quad (8)$$

$$Tf = \frac{1}{2a} \int_{t_0}^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} f(\xi, \tau) d\xi d\tau \quad (9)$$

$$su = \frac{u(x-a(t-t_0), t_0) + u(x+a(t-t_0), t_0)}{2} + \frac{1}{2a} \int_{x-a(t-t_0)}^{x+a(t-t_0)} u_t(\xi, t_0) d\xi \quad (10)$$

Using those operations, the wave equation (5) with the source can be written as:

$$Su = f(x,t) \quad (11)$$

and the generalized Taylor equation indicates directly that the elastic wave as a solution of the corresponding partial differential equation is given by:

$$u = su + Tf \quad (12)$$



as soon as we know the shape and the speed of wave at the time  $t_0$ . The same can be done in the case of a general wave equation (6). It is just necessary to bring the operator  $W$  as the operation  $S$  and select  $s$  and  $T$ . The operations  $s$  and  $T$  have to be defined with the following formulas:

$$T\{f(P,t)\} = \left\{ -\frac{1}{4\pi} \iiint_{\Omega} \frac{Af(P_0,t)}{d(P,P_0)} d\Omega \right\} \quad (13)$$

$$s\{u(P,t)\} = \left\{ \frac{1}{4\pi} \iint_{\sigma} \left[ \frac{1}{d(P,P_0)} A \left( \frac{\partial u}{\partial n} \right) - Au \frac{\partial}{\partial n} \left( \frac{1}{d(P,P_0)} \right) + \right. \right. \\ \left. \left. + \frac{1}{ad(P,P_0)} A \left( \frac{\partial u}{\partial t} \right) \frac{\partial d(P,P_0)}{\partial n} \right] d\sigma \right\} \quad (14)$$

Where  $L^0 = C^1(\Omega \times \langle 0, \infty \rangle)$ ,  $L^1 = C^3(\Omega \times \langle 0, \infty \rangle)$ , operation  $A$  is a substitution for  $t$  values

$t - \frac{d(P,P_0)}{a}$ ,  $a \neq 0$  is a constant characterizing an equivalent system,

$d(P,P_0) = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ ,  $\frac{\partial}{\partial n}$  stands for normal derivative,

$\sigma$  is a boundary of the set  $\Omega$ . Using the generalized Taylor formula, we can obtain:

$$u(x,y,z,t) = u(P,t) = \left\{ \frac{1}{4\pi} \iint_{\sigma} \left[ \frac{1}{d(P,P_0)} A \left( \frac{\partial u}{\partial n} \right) - Au \frac{\partial}{\partial n} \left( \frac{1}{d(P,P_0)} \right) + \right. \right. \\ \left. \left. + \frac{1}{ad(P,P_0)} A \left( \frac{\partial u}{\partial t} \right) \frac{\partial d(P,P_0)}{\partial n} \right] d\sigma \right\} + \left\{ -\frac{1}{4\pi} \iiint_{\Omega} \frac{Af(P_0,t)}{d(P,P_0)} d\Omega \right\} \quad (15)$$

If  $d(P,P_0) > R$  and a source function is limited, then the element  $\frac{1}{4\pi} \iiint_{\Omega} \frac{Af(P_0,t)}{d(P,P_0)} d\Omega$  can be

estimated by  $\frac{\mu(\Omega) \sup|f|}{4\pi R}$  where  $\mu(\Omega)$  is the measure of the set  $\Omega$ . Then, using the formula

for  $u(x,y,z,t) = u(P,t)$  we obtain:

$$u(P,t) \cong \left\{ \frac{1}{4\pi} \iint_{\sigma} \left[ \frac{1}{d(P,P_0)} A \left( \frac{\partial u}{\partial n} \right) - Au \frac{\partial}{\partial n} \left( \frac{1}{d(P,P_0)} \right) + \right. \right. \\ \left. \left. + \frac{1}{ad(P,P_0)} A \left( \frac{\partial u}{\partial t} \right) \frac{\partial d(P,P_0)}{\partial n} \right] d\sigma \right\} \quad (16)$$

The error  $E$  of such estimation meets the following condition  $|E| \leq \frac{\mu(\Omega) \sup|f|}{4\pi R}$ .

Non-classical operational methods enable studies on the continuous relations between the elastic wave propagation and the changes in the source and in the set conditions. When we introduce the result space as the generalized concept of dividing of the space element by operations, which are injections of that space, once can define the generalized Heaviside

operator with  $p = \frac{i_d}{T}$  and express the presented equations for wave propagation as the

functions of the operator  $p$ . Those functions (at this representation of operator calculus) correspond to generalized exponential functions in a selected model. The response of the dynamic system can be found if one knows those exponential functions [12-14].

The practical implementation of the methods described above requires empirical determination of the parameters in the model [15]. The dynamic interactions on other civil engineering constructions called dynamics interactions are transferred through the subsoil. The response of the construction laid on or in the soil may be different from the motion of the subsoil acquired on the soil [16].

The presented wave models along with tuning the system parameters can be used to reduce the vibrations in order to obtain an optimum adjustment of the construction rigidity, vibration parameters and to choose the best vibration and noise eliminators (from the source and environment point of view).

Elastic waves generated by the rolling stock can be used to monitor the technical state of the track, railway foundation, etc. When the emitted field of elastic waves reaches a defect, the inhomogeneity leads to wave reflection in all directions. Therefore, information on the construction with no defects is needed to identify reflected waves due to the defects.

### 3 Conclusion

The elastic waves can be generated intentionally apart from those waves that exist in the nature. The elastic waves can be applied in the technology but are also considered as the side effect of the train traffic, thus having a negative impact on the human being and the environment. The methods of non-classical calculus are feasible for the analysis of many technical problems [17], [18], including the issues associated with elastic wave generation due to the trains. Wave models enable qualitative and quantitative analysis of many problems provided that the model is identified, calibrated and validated. The best approach is to use in situ studies to determine the parameters of the system.

The parameters  $u$  can be determined with numerical or hybrid methods with the use of non-classical operator calculus. The presented models can be applied for the assessment of the structure failure frequency, structure reliability and for the noise reduction.

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