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17–19 May 2018, Zadar, Croatia

# Road and Rail Infrastructure V

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## THEORETICAL-CALCULATION MODEL FOR DETERMINATION ON STRESS AND LOAD-BEARING CAPACITY IN RAIL-TRACK WITH CORROSION

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### Abstract

The researches, connected with the clarification of the influence of the corrosion over the stress and the capability of carrying weight are a key-factor for the correct evaluation of the state of the elements. All the rails are practically subjected to the influence of the atmosphere, so all of them are under the effects of the atmospheric corrosion, which can have three different aspects – reduction of the geometrical characteristic of the section appearance of surface defects (structural changes) and decrease of the mechanical properties. In this study, we present a theoretically calculated module to determine the factual borders of the stress and the possibility of carrying weight of a rail track with corrosion (type “continuous beam”). Dependencies are derived and computational algorithms with calculations and numerical realisation as well.

*Keywords: corrosion, continuous beam, rail-track, stress, load-bearing capacity*

### 1 Introduction

There are numerous railroads in many countries, which have been built more than fifty years ago. Throughout the years, these railroads are subjected to corrosion, irrelevant of whether they are used or not. We already know – there are different types of corrosion (atmospheric, chemical, alkaline, electro-chemical etc.) [1]. The atmospheric corrosion exerts in fact permanent influence over the rails and this leads to the change of the geometrical characteristic of the section, to the appearance of defects on the surface (structural changes) and to the worsening of their mechanical properties [4], [5] etc.

On figure 1 we have shown a rail which has not been used at all for more than 25 years, while on figure 2 we have shown a rail track on which daily (in an interval of 5-10 minutes) there pass motor cars of the subway.



Figure 1 General view

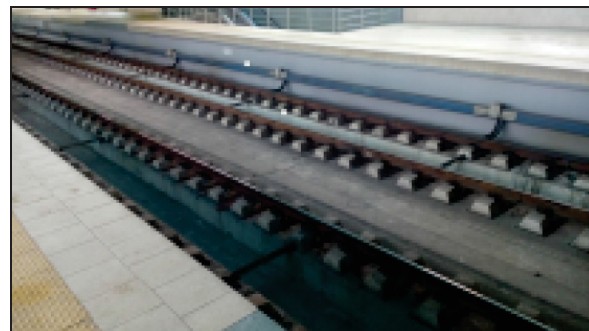


Figure 2 General view over the rail



It is quite obvious that there is corrosion on both types of rails, irrelevant of the fact that one of them is in the open air, while the other is inside a subway station. What is doubtless is that the main factor, influencing them, is the atmospheric corrosion. We have to determine now what the influence is of the corrosion over the stress and the capability of carrying weight of the rail, concerning its maximal use and/or its repeated use with the idea to clarify until which moment it would be capable of exploiting.

## 2 Setting a task

We have observed a steel rail of the type 60 E1, used on sleepers deployed on a 50 cm interval, which has been calculated for a maximal bending moment under weights with already familiar characteristics and density. We would like to see the changes of the stress and the deformities at any given moment of the free development of its corrosion and the moment the border value will be reached will define the factual capability of carrying weight of the corroded rail track.

## 3 A theoretically calculated module

We have observed a 12 meters long rail-track with section of the type 60 E1, Figures 3 and 4.

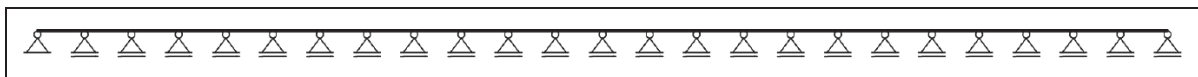


Figure 3 Theoretical static scheme of type “continuous beam”

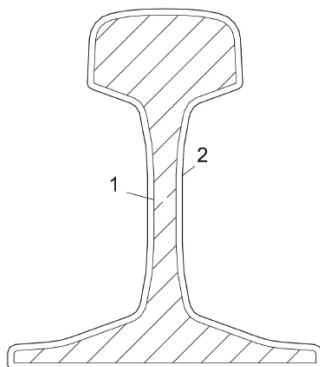


Figure 4 Rail track section (1-non-corroded part, 2-corroded part)

Having used the hypothesis of Bernoulli about the platitude of the sections [3], we have described the deformation as a linear function with two parameters, as other researchers do [7] and [8], so we can express the strain as:

$$\varepsilon_x = \alpha_1 + \alpha_2 \cdot z \quad (1)$$

Then the normal stress is written:

$$\sigma_x = (\alpha_1 + \alpha_2 \cdot z) \cdot E \quad (2)$$

We can determine the stress according to the formula:

$$N(x) = \int_{A_1} \sigma_x dA + \int_{A_2} \sigma_x dA = 0 \quad (3)$$

$$M(x) = \int_{A_1} \sigma_x \cdot z dA + \int_{A_2} \sigma_x \cdot z dA = M(x) \quad (4)$$

The internal stress in any given section should be defined as follows:

$$N(x) = E \cdot \int_{A_1} (\alpha_1 + \alpha_2 \cdot z) dA + E_2(z) \cdot \int_{A_2} (\alpha_1 + \alpha_2 \cdot z) dA = 0 \quad (5)$$

$$M(x) = E \cdot \int_{A_1} (\alpha_1 \cdot z + \alpha_2 \cdot z^2) dA + E_2(z) \cdot \int_{A_2} (\alpha_1 \cdot z + \alpha_2 \cdot z^2) dA \quad (6)$$

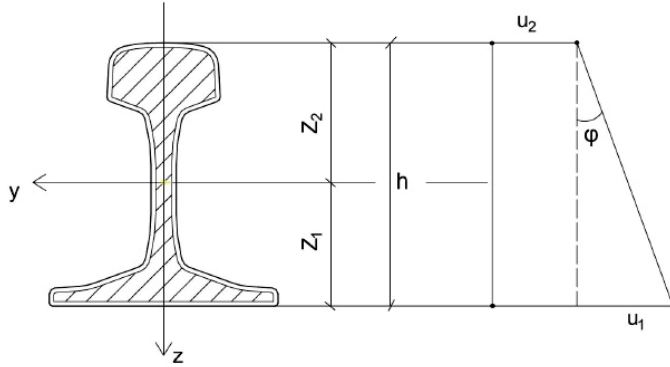


Figure 5 Determine the transformation

We have to determine the transformations – see Figure 5 and we note:

$$u_1 = (\alpha_1 + \alpha_2 \cdot z_1) dx \quad (7)$$

$$u_2 = (\alpha_1 + \alpha_2 \cdot (-z_2)) dx \quad (8)$$

$$-\frac{d\varphi}{dx} = \frac{u_1 - u_2}{h} = \frac{(\alpha_1 + \alpha_2 \cdot z_1 - \alpha_1 - \alpha_2 \cdot (-z_2))}{h} = \frac{\alpha_2 \cdot (z_1 + z_2)}{h} = \alpha_2 \quad (9)$$

The Eq. (5) and Eq. (6) can be presented as a system in the following way:

$$\begin{cases} a_{11} \cdot \alpha_1 + a_{12} \cdot \alpha_2 = b_1 \\ a_{21} \cdot \alpha_1 + a_{22} \cdot \alpha_2 = b_2 \end{cases} \quad (10)$$

Where:

$$a_{11} = E \cdot A_1 + \int_{A_2} E_2(z) \cdot dA ; a_{12} = a_{21} = E \cdot S_{y1} + \int_{A_2} (E_2(z)) \cdot z \cdot dA ; b_1 = 0$$

$$a_{22} = E \cdot J_{y1} + \int_{A_2} (E_2(z)) z^2 \cdot dA ; b_2 = -M(x) ; S_{y1} = \int_{A_2} z \cdot dA ; J_{y1} = \int_{A_2} z^2 \cdot dA$$

- $\alpha_1$  – stress when  $z = 0$ ;
- $\alpha_2$  – according to Eq. (7) and Eq. (9);
- $A_1$  – area of non-corroded part;
- $A_2$  – area of corroded part;

We have to use the basic principle that:

$$w'(x) = \text{tg}\varphi \approx \varphi = \frac{dw}{dx} ; w''(x) = \frac{d\varphi}{dx} = -\alpha_2 \quad (11)$$

Accordingly, there follows:

$$\varphi(x) = -\int \alpha_2 dx = -\alpha_2 \cdot x + C_1 \quad (12)$$

$$w(x) = -\alpha_2 \cdot \int x dx + \int C_1 dx = -\alpha_2 \cdot \frac{x^2}{2} + C_1 \cdot x + C_2 \quad (13)$$

The permanent values  $C_1$  and  $C_2$  are at the borders of the respective conditions, i.e.:  $w(0) = 0$ ;  $C_2 = 0$ ;  $w'(0) = f_1(0)$ ;  $C_1 = f_1(0)$ ;  $w(l) = 0$ .

Having in mind all these dependencies, we can proceed with numerical realisation. The locomotive of the type V100 (diesel) was placed on a corroded rail – Figure 5. We have used the model of Birman [2] (Figure 6) and the forces upon which the rail be influenced are shown on Figure 7. With the help of the theorem of three moments (Clapeyron’s theorem) we have prepared the M-diagram – Figure 8.

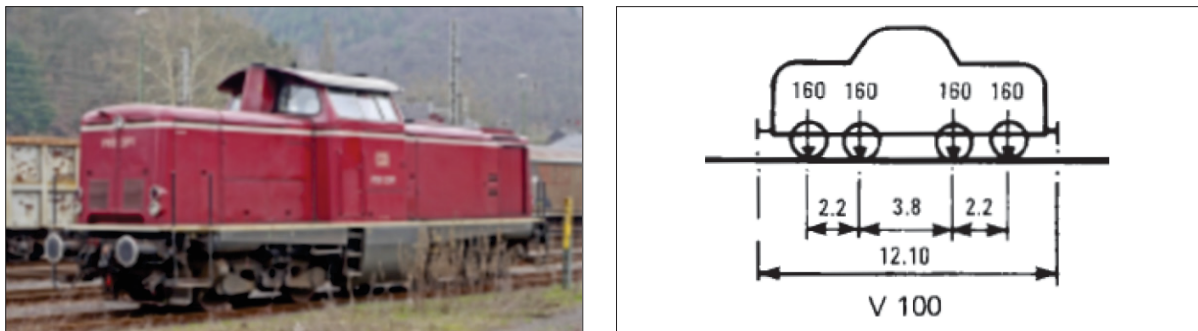


Figure 6 Locomotive V100 on corroded rail (left); model of Birman (forces) [2] (right)

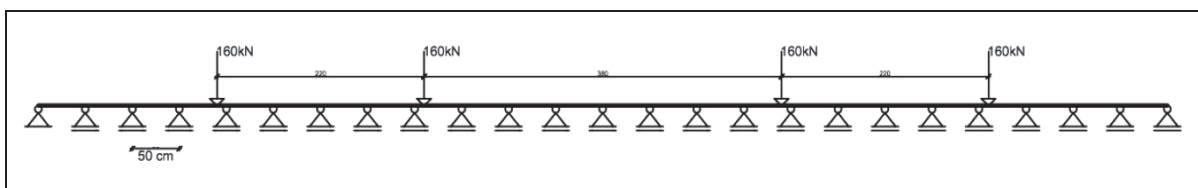


Figure 7 The forces upon which the rail

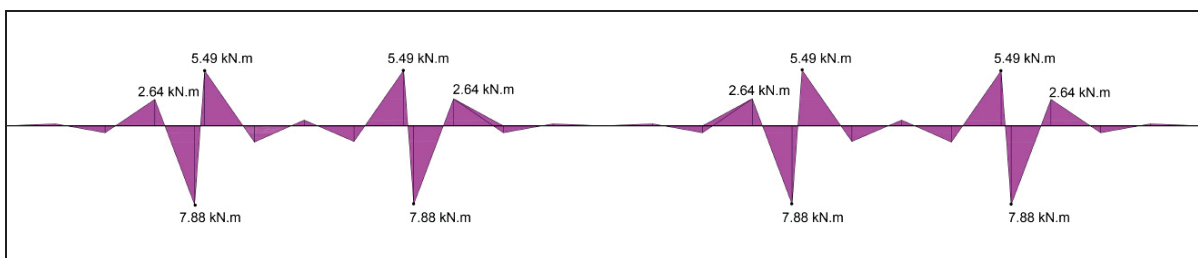


Figure 8 M-diagram



We have used a method of determining the constant value  $C_1$ , for the increase of the value, which involves the application of different values, so we have marked the dependency:

$$w_i = w_{i-1} + \frac{dw}{dx} = w_{i-1} + w'dx \quad (14)$$

i.e. we provide with an initial step  $f=1.10^{-16}$ ,  $dx = 2$  cm (Figure 9 and Figure 10), until the fulfilment of the starting and the bordering conditions of each point on the rail i.e.

$$\begin{cases} a_{11} \cdot \alpha_1 + a_{12} \cdot \alpha_2 - b_1 = 0 \\ a_{21} \cdot \alpha_1 + a_{22} \cdot \alpha_2 - b_2 = 0 \end{cases} \quad (15)$$

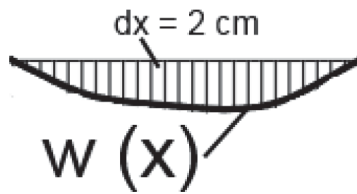


Figure 9  $w(x)$  principal diagram, part  $dx=2$  cm (mesh),  $l=50$  cm

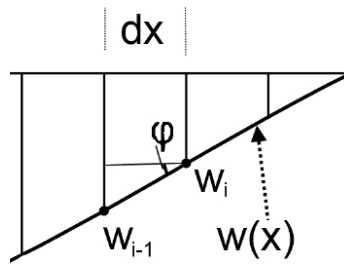


Figure 10  $w(x)$  principal diagram, part  $dx=2$  cm (mesh), step  $f=1.10^{-16}$

Calculation baseline data:

$M = 7.88$  kNm;  $E = 210$  GPa;  $A = 76.70$  cm<sup>2</sup>;  $J_y = 3038.3$  cm<sup>4</sup>;  $S_y = 375.5$  cm<sup>3</sup>;  $G = 81.1$  GPa;  $n = 0.295$  (Poisson's ration)

- 1<sup>st</sup> variant – After time, our rail-track have weight loss ration of corrosion = 0.013 and  $E_2 = 190$  GPa (using data from [6])
- 2<sup>nd</sup> variant – After time, our rail-track have weight loss ration of corrosion = 0.019 and  $E_2 = 185.2$  GPa (using data from [6])

Table 1 Calculation results

	non-corroded track	1-st variant	2-nd variant
$E$ [MPa]	210000	210000	210000
$E_2$ [MPa]	–	190000	185200
$a_1$	$9.994 \times 10^{-5}$	$10.7232 \times 10^{-5}$	$11.0732 \times 10^{-5}$
$a_2$	0	$0.7594 \times 10^{-6}$	$1.0710 \times 10^{-6}$
$s_x$ [MPa]	20.9871	23.8092	25.0736
% increase	–	13.45	19.47

## 4 Conclusions

It is tremendously complicated to determine the influence of the corrosion over the rails, so this requires the search for theoretically based modules to calculate the stress and the capability of carrying weight.

The results show considerable influence of the corrosion over the stress and accordingly over the capability of carrying weight and the eventual term of the overall possible usage of the rails. The theoretical module which is suggested by us is characterised by the improvement of the existing methods of calculation. With this we have achieved greater precision and exactitude in the determination of the stress. It is however considerably labour-consuming, so that makes its use in practice not very applicable.

If there appears a connection between the development of the corrosion in time [9] and the change of the geometrical characteristics and the relative mechanical properties – this model can be applied. It can then help to determine precisely the stress, which would unavoidably appear as a result of the corrosion of the rails.

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## References

- [1] Tzenov, Tz.: Corrosion of materials, 2006, ISBN 978-954-751-071-5
- [2] Brimann, F.: Series Measurements at the rail for determination of the guide forces (in German), ETR Eisenbahntechnische Rundschau, 1966, 15.5: 155-164.
- [3] Timoshenko, S.P., Gere James, M.: Mechanics of materials, 2002, ISBN: 5-9511-0003-8
- [4] Shopov, A., Ganev, R.: Survey on many-years influence atmospheres conditions on strain of the reinforced steel  $\Phi 6.5$  (A-I) for once more using, Proceeding of International Jubilee Scientific Conference “75<sup>th</sup> Anniversary of UACEG”, ISSN 2534-9759 (in press)
- [5] Xu, G., et al.: Mechanical properties of steel corrosion products in reinforced concrete, Materials and Structures, 2017, 50.2: 115, doi: 10.1617/s11527-016-0985-3
- [6] Gao, D., Zhao, Z.: Experimental study on mechanical properties degradation of TP110TS tube steel in high H<sub>2</sub>S corrosive environment, Computers Materials and Continua, 2011, 26.2: 157-166.
- [7] An, L., et al.: A simple approach to determine mechanical properties of rust layer on steel substrate, Proceeding of the Japan Concrete Institute, 2014, 36.1: 1264-1269.
- [8] An, L., et al.: Determination of young's modulus of rust-layer by bending experiment, Proceeding of the Japan Concrete Institute, 2013, 35.1: 1117-1122.
- [9] Svintradze, D., Pidaparti, R.: A theoretical model for metal corrosion degradation. International Journal of Corrosion, 2010, doi: 10.1155/2010/279540