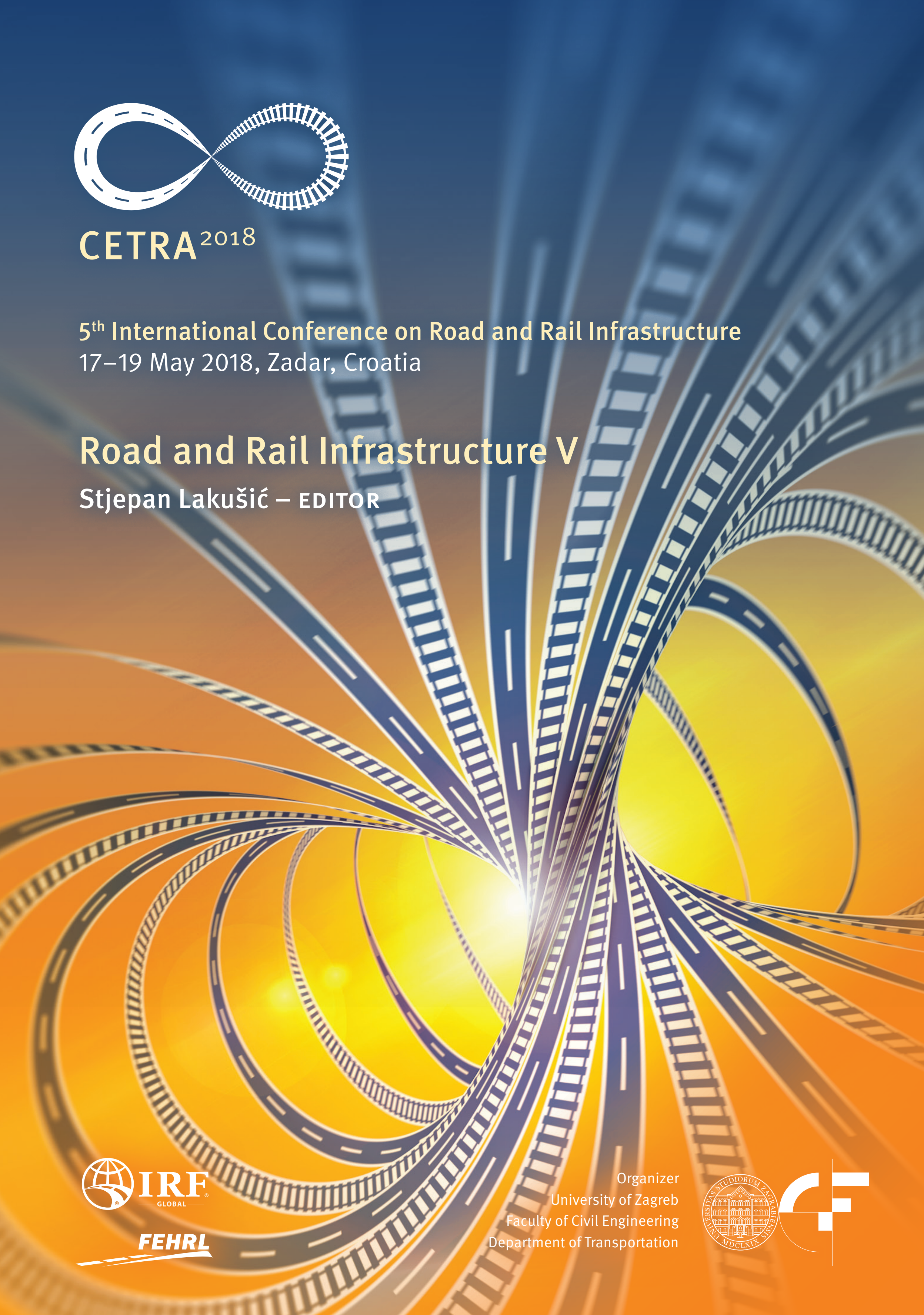

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5th International Conference on Road and Rail Infrastructure
17–19 May 2018, Zadar, Croatia

Road and Rail Infrastructure V

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Road and Rail Infrastructure V

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NUMERICAL SIMULATIONS OF THE RAILWAY BALLAST

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Abstract

Degradation of the railway ballast occurs in the classical railway structure after a certain time of use. Several factors influence the life span of the ballast, such as quality of the stone material, quality of subgrade, sleepers, loadings of the structure, etc. In order to determine and predict the behaviour of the ballast, we need to define the load transfer path and interaction of the ballast particles. The numerical simulations are usually carried out in order to determine the characteristics of the loaded structure. Discrete element method (DEM) has several advantages in use for research of the granular material such as railway ballast. Several simplifications of the railway system in the numerical models affect the accuracy of the simulation results in different ways. This review paper of the numerical simulations on ballast behaviour, presents that results of the simulations are in a good agreement with experimental investigation results, therefore the use of numerical models for prediction of railway ballast behaviour is justified.

Keywords: railway ballast, numerical modelling, discrete element method, analysis

1 Introduction

Most railways in the world are running on the ballasted track, [1]. Ballasted track usually consists of rails, sleepers, fastening system and ballast, as in the Figure 1. All parts of the structure are subject to load influences induced by railway vehicles, which affects its lifespan. Quality of the railway track depends on the quality of all its segments, so the quality of each part of the structure must be satisfied in order to meet requested quality of the track. With aging of the structure, the frequency of tamping must rise, in order to keep the track quality that is required by track design project, [1]. This is happening because of the reduction of ballast tamping efficiency during ballast service life.
Ballast behaviour is not fully investigated, therefore many efforts are made in order to find out more about load transmission through structure. The behaviour of the ballast grains under loading is the subject of interest. Maintenance costs in Croatia exceed 30% of the total amount invested in railways, more precisely investment of HRK 45 million was planned in the year 2017, according to National Rail Infrastructure Program for the period 2016 to 2020, [2].

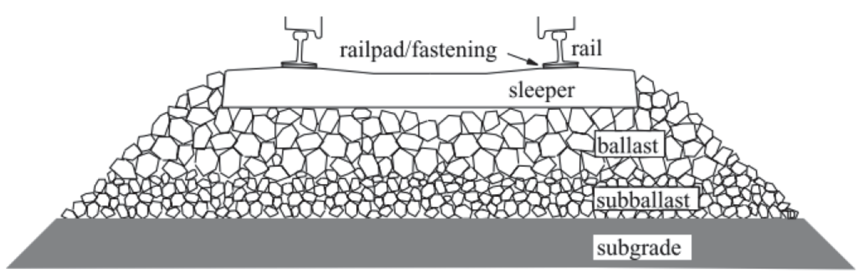

Figure 1 Components of the railway track [3]

Usually, maintenance and renewal of ballast is scheduled in terms of its performance, [1]. Proper design of the ballast characteristics, with specific conditions of investigation field, can result in longer lifespan of the ballast. In the ballast assembly every particle has to satisfy the needed characteristics, but the ballast layer is affected not by single element, but by the cumulative effect of the mixed aggregate characteristics, [4].

2 Railway ballast structure

2.1 Ballast characteristics

Ballast is crushed granular material in the substructure in which the sleepers are embedded, [4]. It is placed on top of the track subgrade in order to support the track structure, [3]. The stone particles of which ballast is consisted should be angular, crushed, hard stones and rocks, uniformly graded and free from dust and dirt, [4]. According to EN 1345:2002, the size of the ballast particles is from 31.5 mm to 63 mm, [5]. Thickness of ballast is 30 to 50 cm, wherein the depth around sleeper ends is higher than below rails, [3]. Its behaviour influences many parts of the railway track. According to [4] five most important ballast functions are:

1) Sufficient bearing capacity to resist vertical, lateral and longitudinal forces that are applied to retain track in its required position,
2) Improve elastic characteristics and characteristics of energy absorption of the track,
3) Provide immediate drainage of water falling on the track,
4) Facilitate maintenance surfacing and lining operations (to adjust track geometry) by the ability to rearrange ballast particles with tamping,
5) Reduce pressures from the sleeper bearing area to acceptable stress levels for the underlying material.

The shape of the ballast particle is complex, irregular and varies within the particles in the ballast layer of each of the individual rail lines. Therefore, its shape requires significant efforts in order to define grain characteristics. Many laboratory tests are usually conducted to obtain the grain characteristics such as flakiness, elongation, sphericity, roundness (angularity), surface texture and to remove the grains of insufficient quality [6]. In order to quantify the complex behaviour and to determine the quality of the ballast, mostly used experimental work is the cyclic triaxial test. It defines the mechanical behaviour of ballast under a large number of passing train axels, [7]. Another test, introduced by Norman and Selig, [6], is ballast box test, where sleeper settlement, ballast breakage and abrasion, changes of stiffness and density of ballast, and horizontal residual stresses in the ballast were investigated. Effects of the train loading and tamping can also be recorded in the box test, introduced by McDowell et al, [8]. The set-up of the box test on the Figure 2, [8], shows basic parts of the test.

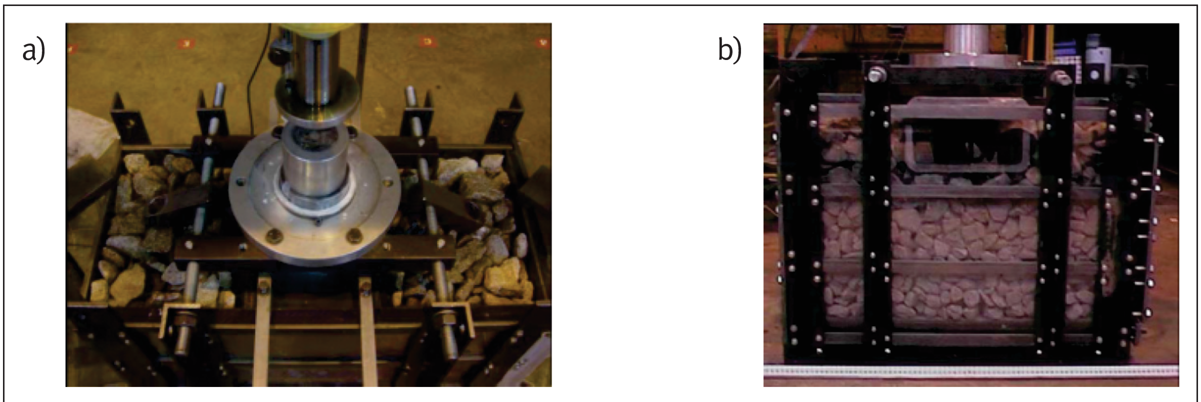

Figure 2 Ballast box test: a) view from the top and b) front view, [8, 9]

Several tests on the railway ballast characteristics and behaviour can describe more precisely the non-linear and complex behaviour of the ballast.

2.2 Ballast loading and degradation

Ballast material should be resistant to chemical (for example coal dust fines), mechanical (loadings, static and dynamic) and environmental (weather) effects. Due to high frequency vibrations and uneven settlements, ballasted track has uneven settlements with defects in different track position, irregular ballast damage that affects the rail condition. When the settlement of the track exceeds 20 mm, which occurs after 30-60 million tons of service, general track maintenance is required. The change of the entire ballast layer happens after 30 years of service, [10]. Since the particles have sharp edges, the contact area with the neighbouring particle will be very small, causing large contact stress which lead to fracture and abrasion while loading. The fines forming during abrasion and fracture fill during time the bottom of the ballast layer, [11, 12]. When fines are combined water (rain, snow), fines will act as a lubricant, causing ballast material abrasion and pumping under the sleepers, [11].

2.3 Ballast maintenance

Track vertical and lateral alignment during years of exploitation changes its accuracy and the settlement appears. During loading, alignment experiences elastic displacements and after a certain number of cycles, residual deformation appears. It affects the quality of train ride, affecting even riding speed, depends on the amount of derailment, [4]. During ballast service life, the distance between sleepers and ballast increases, which can lead to insufficient load transfer in the railway track. Therefore, it has to be repaired, usually with maintenance process of tamping. Tamping is the most effective way of correcting geometry deficiencies, it is the process of rearranging of the stone particles, with rearrangement tools; Fig. 3, in the ballast layer to fill voids under the sleepers, [4, 13]. The result of tamping is adjusting track to desired geometry.

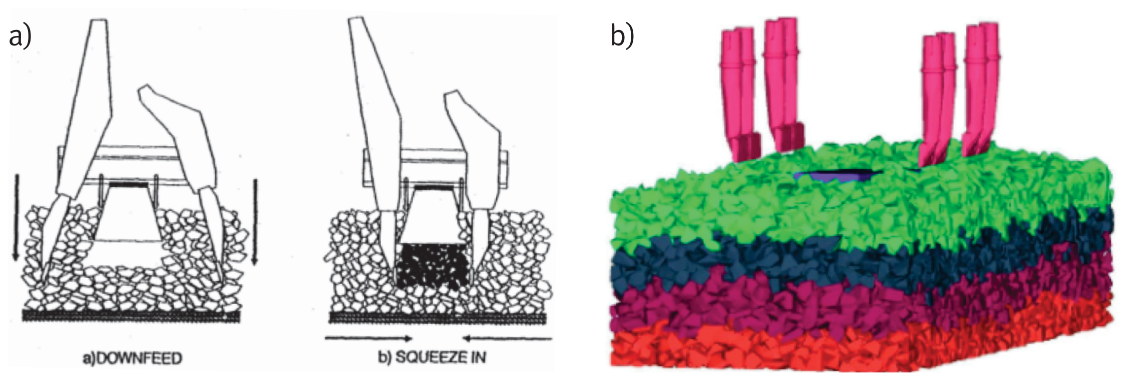

Figure 3 Process of tamping; a) movements of tamping tools, [4], b) numerical model of the ballast tamping, [14]

The amount of the fouled ballast in the ballast layer increases with increase in the load cycles, therefore, the process of tamping should be performed more often (Figure 4 left), [4]. When the track structure is new, notable endowment comes from the subbalast and subgrade, as in Figure 4 right.

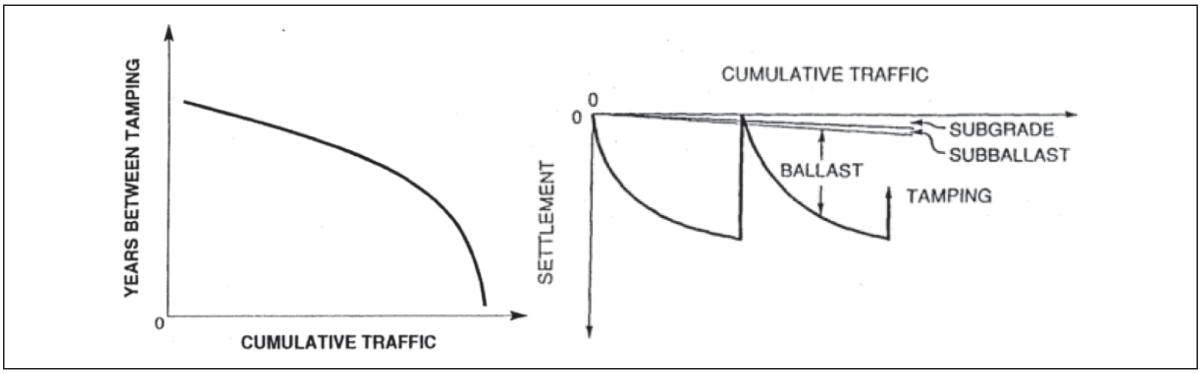

Figure 4 Reduction of the tamping efficiency and influence of the substructure on the settlement, [4]

During tamping, because of the rearrangement of the particles, new particles contact points are produced which causes increased breakage under additional traffic loads, [4]. One of the first steps in providing efficient maintenance of the track is to systematize and segment the railway lines network. According to the real condition of different segments, it is possible to evaluate and rank the segments of the track for maintenance, [15].

3 Numerical modelling of the railway ballast

3.1 Basic Setup

Simple numerical model, presented in [16], Figure 5, was made in the computer program called "Rail", developed on the Delft University of Technology, for qualifying dynamic behaviour of railway track, [16]. Different characteristics of the components, such as rail, rail pad, sleeper and ballast can be set. Ballast layer is defined as the continuous layer with constant value of damping and stiffness. The model is calculated with finite element method. Due to the discrete nature of the particles, ballast layer behaviour is not determined sufficiently precise. Therefore, the Discrete element method (DEM) is introduced for numerical modelling of such particles, by Cundal and Strack, [17]. Interpretation of tests on real granular media is difficult because the stresses inside the sample must be estimated from the boundary conditions. In DEM, the equilibrium contact forces and displacements of discs that present particles, are found through a series of calculations tracing the movements of the individual particles, [17].

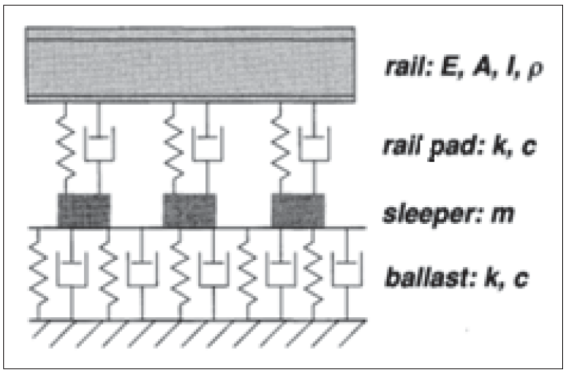

Figure 5 Schematisation of the ballasted track with characteristics of its parts, [16]

3.2 Discrete element method

One of the main problems numerical simulations of the railway ballast are dealing with is fouling of the ballast. Ballast fouling is the process by which the voids between particles become filled with fouling material. The fouling material is derived from ballast deterioration from repeated loading and tamping and from sources external to the ballast, [18]. It is important to find out when the ballast will reach critical rate of contamination with fine particles to affect the behaviour of the track. Different critical phases of ballast fouling are presented on the Figure 6, [19].

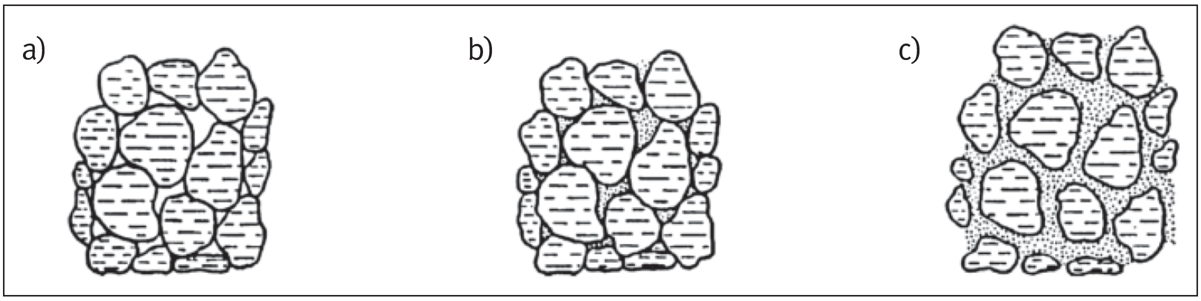

Figure 6 Phases of the ballast fouling: a) Clean ballast, b) Partially fouled ballast, c) heavily fouled ballast, [19]

3.3 Particle shape

The influence of the particle shape in the DEM is wide, including the physical state of the assembly and particle interaction, [6]. Particle interaction includes interparticle friction and contact forces between particles. Ballast assemblies in the numerical simulations are usually made with spheres, clumps of spheres or polyhedral particles (Figure 7a).

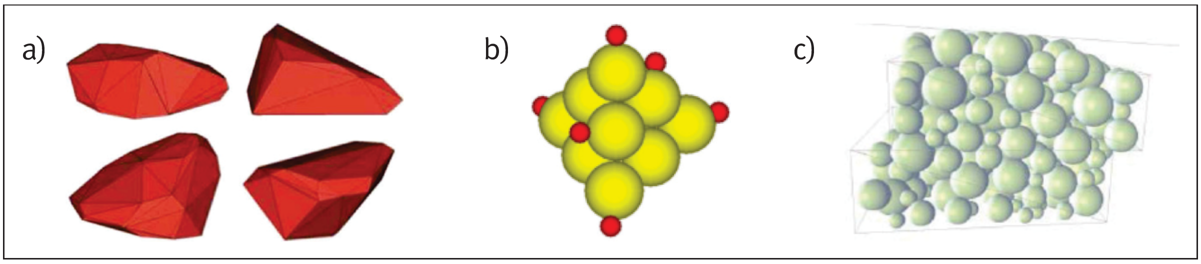

Figure 7 a) Polyhedral shapes, [20], b) spheres, [21] c) numerical ballast particle with its real counterpart, [22]

Shapes of the particles can be obtained from the three orthogonal pictures of the real ballast particles, [22]. That way, realistic polyhedral particle shapes can be obtained, Figure 7b. Different shape particles, as in the [22] with spheres and polyhedral particles, in the triaxial tests, affect the accuracy of the ballast performance. Different grain shapes during numerical simulation of the shear tests, conducted by Dubina & Eliáš, [23], show dissimilar behaviour in terms of vertical displacements, as in the Figure 8.

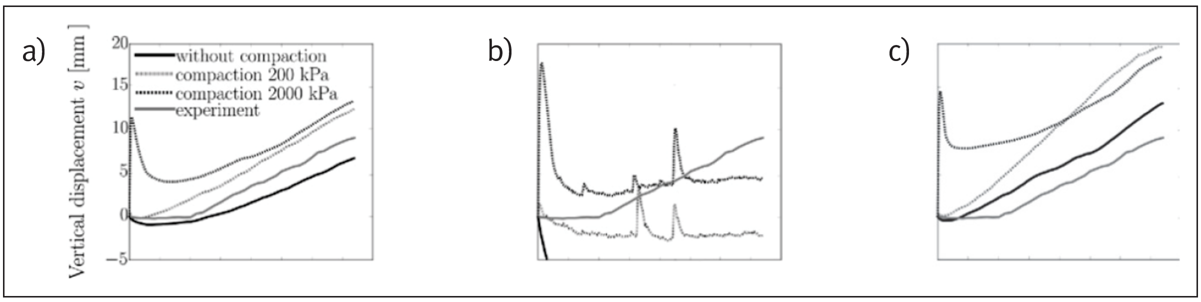

Figure 8 Results of vertical displacement in shear tests with particles of different shapes: a) spheres with identical R, b) polyhedrons, c) spheres with variable R, [23]

Numerical simulation of the box tests, conducted in the box of dimensions 700mm x 300 mm x 450 mm, were performed in computer program PFC^{3D}, [24]. Three boxes were filled with spheres, clumps with larger maximum diameter and spheres with smaller maximum diameter, respectively, Figure 9. The ballast was compacted in order to remove any large voids between particles (Figure 9), [24], and boxes were exposed to cyclic loading through the sleeper placed on top of the box, Figure 10. Three types of elements of different complexity show significant differencies in the results, giving a better contact forces distribution and a better load-deformation response of the clumps, [24].

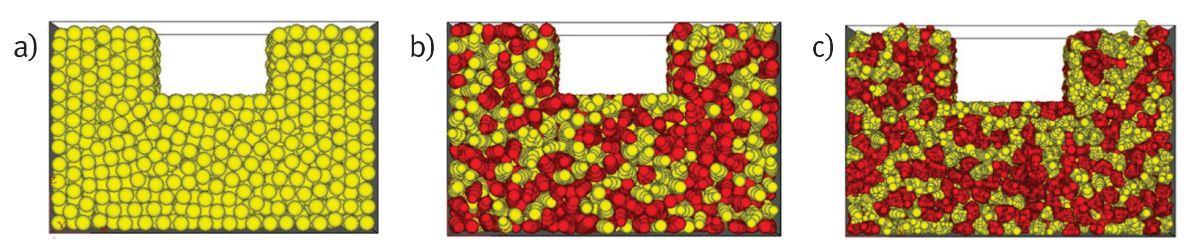

Figure 9 Simulation of the box tests on different elements: a) spheres, b) clumps, c) clumps with smaller diameter, [24]

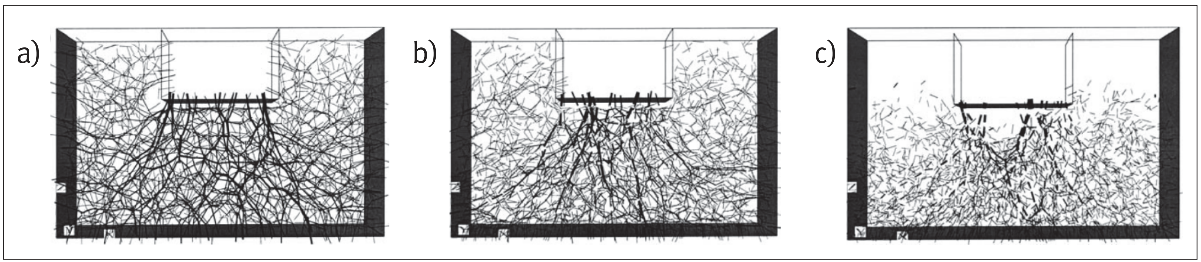

Figure 10 Contact forces after the compaction of the assembly for: a) spheres, b) clumps, c) clumps with smaller diameter, [24]

3.4 Contact forces

Ballast grains in the track are moving due to the external loads from railway vehicles. Loads, during shifting to the subgrade, transfer through the ballast layer over contact points between particles. Therefore, the contact forces, appearing in the contact points, play the key role in the force transportation of the track. Loading forces in the ballast layer lose the intensity throughout friction between particles, due to rough surface of the grains. Consequently, contact laws that define the interaction between grains, significantly affect the performance and accuracy of the simulation. The interaction of the particles in numerical simulation of the ballast assembly is determined as in the Figure 11, where nodes represent centre of the particles and are connected with springs of desired characteristics. Observation of the contact force distribution shows intense strain in the area underneath the sleepers, which is in agreement with the laboratory tests, presented of the Figure 12, [26].

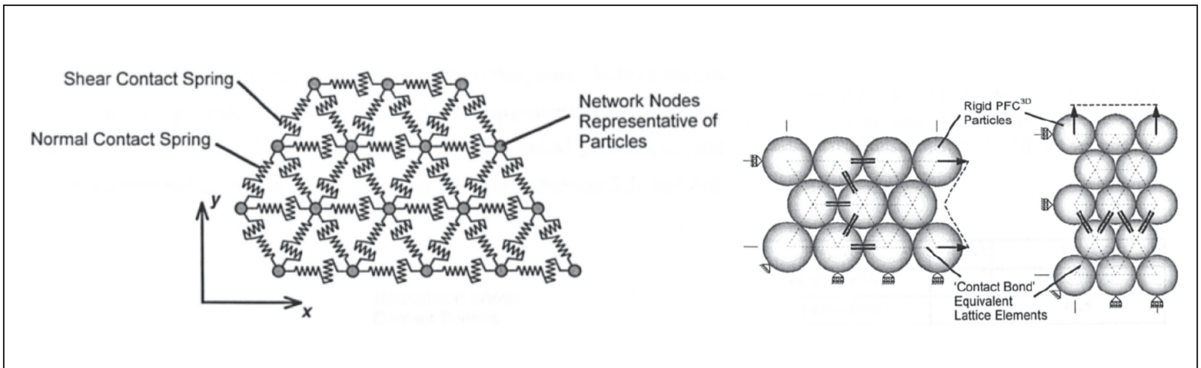

Figure 11 Particles assembly with spring network, [25]

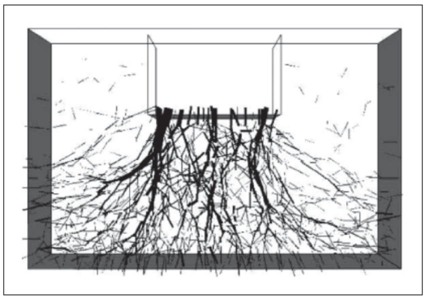

Figure 12 Contact force distribution during cyclic loading (breakage of the particles is allowed), [26]

3.5 Particle breakage

Dynamic loads influence particle rearrangement, which causes settling of the railway track, [27]. Saussine et al., [28], based on the good agreement of the results of the numerical simulations in 2D and experimental results, proved that it is possible to evaluate the settlement of the angular particle assembly using numerical simulations. With the development of numerical simulations of ballast, different ways of solving the problem of particles breakage are introduced, [27]. Particle breakage appears when the contact force exceeds given permitted value. One of the solutions consider particles as porous agglomerates that are built by bonding smaller particles. During the simulations, agglomerates can disaggregate, as in [6, 29]. Another solution of particle breakage is to replace the particles with equivalent group of smaller particles, when fulfilling a predefined failure criterion, [30, 31]. According to [27], breakage of the particles gives more realistic results. Permanent settlement in the ballast layer significantly increases with considering of the particle breakage, [27, 26], and is approximately proportional to the logarithm of the number of cycles of load, when the number of cycles is small, [26].
Ngo et al., [33], in numerical simulation of the large scale simulation testing of the track process showed that an increase in the subgrade stiffness results in an increased number of broken bonds, thus affecting the settling of the ballast layer.

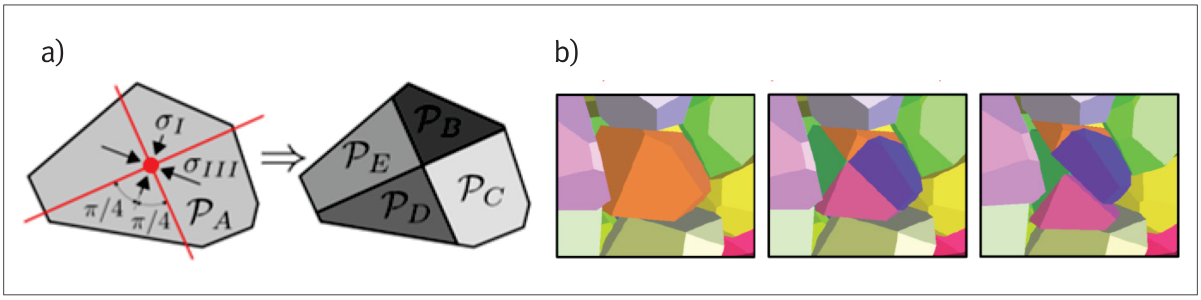

Figure 13 a) particle breakage into 4 elements, b) numerical simulation of breakage, [32]

3.6 Diversity of use

Ballast has significant influence on the spreading of the loading through the railway structure. Likewise, other parts of the structure influence the ballast with theirs behaviours. Gao et al. recently investigated the influence of the movements of the sleepers on the ballast. Research shows that angular accelerations of sleepers with sleeper-ballast gap could generate extra moment up to 5 kNm, [34]. That way it may increase the ballast breakage, interfering service life of track. Numerical model of the maintenance process of tamping can show, [14], the optimum intervals for tamping during ballast service life. Another implementation of the numerical simulations with ballast is investigation of the polymer implementation in the ballast assembly, such as geogrid and elastic pads. Geogrid reinforced ballast has better behaviour in terms of shear strength and decrease dilation due to interlocking between the ballast and the geogrid, but disadvantage is the filling of the voids between particles, which affects the drainage, [35, 36]. The DEM numerical model of the ballast structure with the under sleeper pads, which have been previously experimentally shown to have a beneficial effect on vibration reduction, [37], proves the advantages of its usage. The evidence provided by DEM shows that application of under sleeper pad allows more particles to be in contact with the pad, and causes these particles to transfer a larger lateral load to the adjacent ballast but a smaller vertical load beneath the sleeper, [38].

4 Conclusions

In order to study ballast performance in the classical railway track, numerical simulations using discrete element method are usually implemented. Simplifications implemented into the models do not significantly affect the results of the tests, which are well correlated with the experimental results. Researchers use two different shapes of particles: spheres and polyhedral particles. Spheres are most common in the ballast assemblies, giving precise results and not requiring great computing costs. Polyhedral shapes give even better results, but the computing time is much longer. Contact forces in the particle assembly has great influence on the behaviour, allowing particle interaction and even breakage of the bonds between particles. The performance of the ballast layer can be optimized by controlling particle characteristics. Numerical simulations allow us to deeper understand the behaviour of the ballast and to predict and plan optimum maintenance work. Several different influences on the ballast can be numerically computed, such as tamping, triaxial tests, ballast box tests, etc. The permanent settling of the ballast layer increases when particles are allowed to break. Numerical simulations give results that are in a good agreements with experimental results, therefore it justifies the use of numerical simulations for dealing with railway ballast problems.

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