CETRA[2018]

5th International Conference on Road and Rail Infrastructure
17–19 May 2018, Zadar, Croatia

Road and Rail Infrastructure V

Stjepan Lakušić – EDITOR

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CETRA[2018]
5th International Conference on Road and Rail Infrastructure
17–19 May 2018, Zadar, Croatia

TITLE
Road and Rail Infrastructure V, Proceedings of the Conference CETRA 2018

EDITED BY
Stjepan Lakušić

ISSN
1848-9850

ISBN
978-953-8168-25-3

DOI
10.5592/CO/CETRA.2018

PUBLISHED BY
Department of Transportation
Faculty of Civil Engineering
University of Zagreb
Kačićeva 26, 10000 Zagreb, Croatia

DESIGN, LAYOUT & COVER PAGE
minimum d.o.o.
Marko Uremović · Matej Korlaet

PRINTED IN ZAGREB, CROATIA BY
“Tiskara Zelina”, May 2018

COPIES
500

Zagreb, May 2018.

Proceedings of the
5th International Conference on Road and Rail Infrastructures – CETRA 2018
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A NUMERICAL METHOD FOR ESTIMATING THE DEFLECTIONS OF BALLASTED RAILWAY TRACKS

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Abstract

This paper presents a method to estimate the deflection in ballasted railway tracks and compares the results with field measurements, performed by other researchers. The influence of different variables on deflection estimation such as loading, gauge, rails, sleepers, layers' thickness and resilient modulus was parameterized through 768 simulations of representative layered structures using the finite-element software ABAQUS. It is not about conventional numerical simulations, but simulations implementing a theoretical-empirical model developed in Brazil, with Brazilian soils from different regions, by Guimarães (2009) in your PhD thesis. For this implementation, a internal subroutine named UMAT, from ABAQUS, was developed in Fortran. With the deflections results from each simulated track, a matrix with 768x7 elements was formed and solved with a MATLAB program. As a result, an equation was defined to estimate deflections of real railway tracks similar to those simulated. Comparisons among the numerical simulation results and field tests show that the proposed method can be applied to predict vertical displacement of railway tracks.

Keywords: railway, tracks, ABAQUS, UMAT, deflections

1 Introduction

In Brazil, the deflection in railways has been measured using the Benkelman Beam or specific variations, positioning its tip on the base of the rail, as shown at Fig. 1, measuring the vertical displacement under the axle load and at other distant points from the axle, if necessary. One of the first measures of this type in Brazil was carried out by Spada [1], using a dial gauge or LVDT with a millimetric precision.
In this aspect, it is necessary to develop new mathematical methods for deflection estimation, based on adequate theoretical models, such as traditionally methods of Zimmermann [5] and Talbot [6], mentioned by Chramm [7], Queiroz [8], Sadeghi [9], Silva [10], Steffler [11], among others. In newly constructed railways or under maintenance, establishing the deflection parameter from specific numerical simulations or using the proposed equation, the track condition can be evaluated.

DOI: https://doi.org/10.5592/CO/CETRA.2018.963

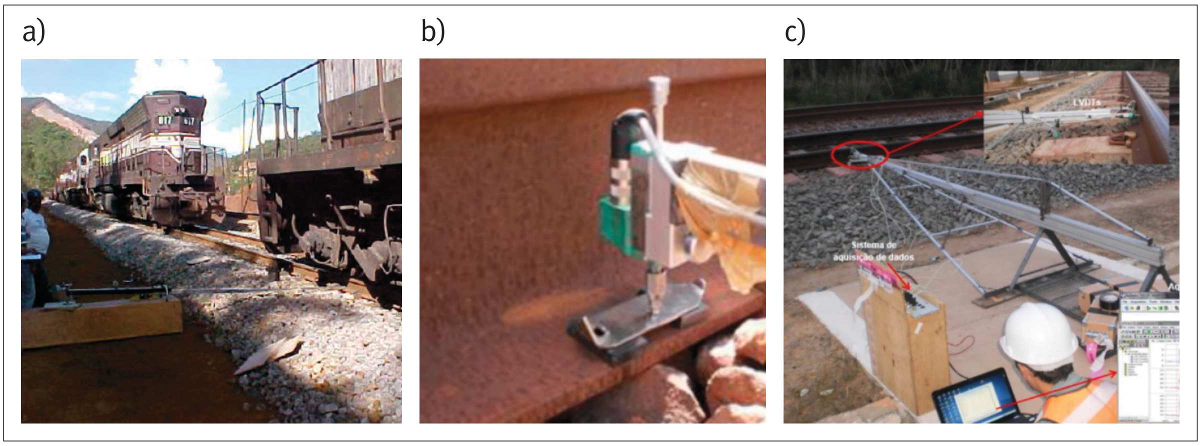

Figure 1 Examples of rail track deflection measures conducted by Brazilian researchers: a) Fernandes [2]; b) Merheb [3]; c) Costa [4]

2 Proposed method for deflection estimation

2.1 The Guimarães' (2009) method

According to Guimarães [12] and Klincevicius [13], there are three main techniques for modeling the deformation of soils used for track construction: using a relationship with the number of loadings, such as the Monismith's et al. model [14]; analyzing the material's stress state; using the Shakedown theory, analysing the elastoplastic behavior of the materials submitted to loading cycles. The parameters of the Monismith's model may suffer changes with the increase of loading cycles and aiming to improve the formulation, using the number of loading cycles, the stress state and Shakedown theory, Guimarães [12] proposed the Eq. (1), that better describes the conditions of soil deformation.

$$\varepsilon_p(\%) = \Psi_1 \left(\frac{\sigma_3}{\sigma_{ref}}\right)^{\Psi_2} \left(\frac{\sigma_d}{\sigma_{ref}}\right)^{\Psi_3} N^{\Psi_4} \quad (1)$$

Where: Ψ_1, Ψ_2, Ψ_3, Ψ_4 are experimental parameters according; N is the number of loading cycles; σ_{ref} is the reference stress, considered equal to atmospheric pressure (0.1 MPa); σ_3 is the confining stress (MPa); σ_d is the deviator stress (MPa).

2.2 The simulations performed

Using appropriate elastic and elastoplastic models, 768 tracks configurations (48 tracks with 16 variations of material properties and loads) were simulated using the Finite Element Method (FEM). The Guimarães' model, specifically developed for Brazilian soils, was used in subgrade and sub-ballast simulations, implemented by a subroutine called UMAT (User subroutine to define a material's mechanical behavior), programmed in FORTRAN language and compiled by ABAQUS 2016 software. Drucker-Prager criterion was considered for ballast, in accordance with Profillidis [15] and after comparisons with methods proposed by Indraratna et al. [16]. Only rails, sleepers and fastenings were represented by the linear elastic model, given the magnitude of the stresses acting compared to those elements' strength. It is known that in railway tracks with a certain time of use, these elements suffer severe wear, which was not considered. The convergence study resulted in the finite element quantities is presented in Table 1 and an example of simulated railway track is presented in Fig. 2.

Table 1 Ref. number of finite elements for rail track simulation according to the convergence study of a 2D model

Track component	Number of finite elements used
Rails	186 and linear elastic model
Support devices	152 and linear elastic model
Sleeper	18,068 and linear elastic model
Ballast	9,618 and linear elastic model with Drucker-Prager yield criterion
Sub-ballast	258 and Guimarães' model, considering graded gravel with 150,000 loading cycles
Subgrade	3,698 and Guimarães' model, considering clay or sand with 150,000 loading cycles
All elements used were CPE3 type, triangular in flat deformation state, with 3 nodes and 6 degrees of freedom.	

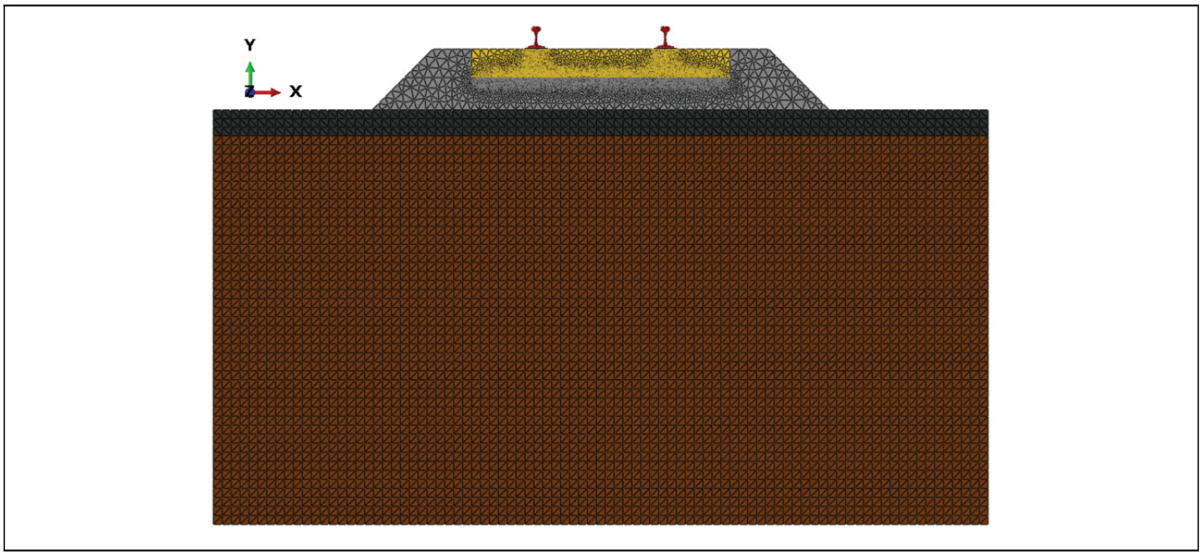

Figure 2 Example of simulated track

2.3 The formulated matrix and the proposed equation

From the simulation deflection results, the matrix system presented in Eq. (2) was created, correlating the intervening variables, defined based on rail track's relevant properties. The matrix [A] which correlates influence factors is known and vector [C] in the right side of the equality is formed from the deflections found in the numerical simulations. It is necessary to calculate the vector product of unknown $[a,...,g]_{7\times 1}$, called [X], and the software MATLAB R2016a was used due to mathematical complexity. Solving the matrix system [A] · [X] = [C], the constants needed to form the generic equation are obtained.

$$\begin{bmatrix} \sqrt{\frac{Q_{11}}{U_1}} & \sqrt[4]{\frac{I_{T12}\cdot E_{T12}}{U_1}} & \sqrt[4]{\frac{I_{D13}\cdot E_{D13}}{U_1}} & B_{14} & \frac{L_{15}\cdot ML_{15}}{U_1} & \frac{SL_{16}\cdot MSL_{16}}{U_1} & \frac{SU_{17}\cdot MSU_{17}}{U_1} \\ \sqrt{\frac{Q_{21}}{U_2}} & \sqrt[4]{\frac{I_{T22}\cdot E_{T22}}{U_2}} & \sqrt[4]{\frac{I_{D23}\cdot E_{D23}}{U_2}} & B_{24} & \frac{L_{25}\cdot ML_{25}}{U_2} & \frac{SL_{26}\cdot MSL_{26}}{U_2} & \frac{SU_{27}\cdot MSU_{27}}{U_2} \\ \sqrt{\frac{Q_{31}}{U_3}} & \sqrt[4]{\frac{I_{T32}\cdot E_{T32}}{U_3}} & \sqrt[4]{\frac{I_{D33}\cdot E_{D33}}{U_3}} & B_{34} & \frac{L_{35}\cdot ML_{35}}{U_3} & \frac{SL_{36}\cdot MSL_{36}}{U_3} & \frac{SU_{37}\cdot MSU_{37}}{U_3} \\ \sqrt{\frac{Q_{n1}}{U_n}} & \sqrt[4]{\frac{I_{Tn2}\cdot E_{Tn2}}{U_n}} & \sqrt[4]{\frac{I_{Dn3}\cdot E_{Dn3}}{U_n}} & B_{44} & \frac{L_{n5}\cdot ML_{n5}}{U_n} & \frac{SL_{n6}\cdot MSL_{n6}}{U_n} & \frac{SU_{n7}\cdot MSU_{n7}}{U_n} \end{bmatrix}_{n\times 7} \cdot \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{bmatrix}_{7\times 1} = \begin{bmatrix} def_1 \\ def_2 \\ def_3 \\ def_n \end{bmatrix}_{n\times 1} \qquad (2)$$

$$[A]\cdot[X]=[C]$$

Where: U_n is track modulus of n^{th} simulation $[F][L]^{-2}$; Q is applied load. In the simulations carried out, loading Q was considered on the same axis that passes through the center of the

sleeper. [F]; E_T is the elastic modulus of de rail $[F][L]^{-2}$; I_T is the moment of inertia of the rail $[L]^4$; E_D is the elastic modulus of the sleeper $[F][L]^{-2}$; I_D is the moment of inertia of the sleeper $[L]^4$; B is the gauge [L]; L is the thickness of ballast layer [L]; ML is the resilient modulus of ballast, preferably obtained in the laboratory $[F][L]^{-2}$; SL is the thickness of the sub-ballast layer [L]; MSL is the resilient modulus of sub-ballast, preferably obtained in the laboratory $[F][L]^{-2}$; SU is the thickness of subgrade, it is recommended to consider from 2 to 5 m, according to the material [L]; MSU is the resilient modulus of the subgrade, preferably obtained in the laboratory $[F][L]^{-2}$; a, b, c, d, e, f, g is the constants to be determined from the matrix system; def_n is the deflections found in rail top in the position under load in each simulation performed.
As deflection is measured in length units, considering the track modulus (U) as the two-dimentional stiffness parameter of railway track, according to Teixeira [18], it was chosen as normalizing variable of each intervening parcel, transforming it into the same deflection unit. According to Raymond [19] the track modulus is in the range of 34 to 69 MPa. According to Selig and Li [20] an increase in ballast thickness can leads to a growth of the U.

3 Results and comparison with field measurements

After 768 simulations and solving the matrix system it was possible to develop the generic equation (3), used to estimate railway track deflection with properties within the ranges applied in the simulations.

$$\left| \begin{array}{c} 2{,}3640\sqrt{\frac{Q}{U}} - 0{,}2284\sqrt[4]{\frac{I_T}{U}} + 0{,}0001\sqrt[4]{\frac{I_D \cdot E_D}{U}} + 0{,}0335\,B \\ + 0{,}0034\frac{L \cdot ML}{U} + 0{,}0075\frac{SL \cdot MSL}{U} + 0{,}0049\frac{MSU}{U} \end{array} \right| = \text{def} \qquad (3)$$

Where: U is track modulus between 10.56 and 123.96 MPa; Q is the static load of one wheel from 125 kN to 200 kN; I_T is the moment of inertia of the rail, between 2730.5 cm^4 (TR-57) and 3850.1 cm^4 (TR-68); E_D is the elastic modulus of the sleeper: 205 GPa for steel; 33 GPa for concrete; 13 GPa for wood. I_D is moment of inertia of the sleeper: 270 cm^4 for steel; 22183.33 cm^4 for concrete; 9826 cm^4 for wood. B is the gauge between 1 m and 1.6 m; L is thickness of the ballast layer between 0.25 m and 0.40 m; ML is the resilient modulus of ballast between 300 MPa and 500 MPa; SL is the thickness of the sub-ballast layer between 0.10 m and 0.20 m; MSL is the resilient modulus of the subgrade between 200 MPa and 300 MPa; MSU is the resilient modulus of the subgrade between 150 MPa and 250 MPa; def is the rail top deflection in mm.
A rectified equation is also proposed, where it is subtracted 1.9 mm, the average of differences between the deflections estimated by equation (3) and the simulated deflections, aiming to make the average of the deflections obtained by equation equal to the average of the simulated deflections (more details are not being presented in this paper due its limitation, but can be obtained in [21]).
Comparison between the formulated equations and deflections measured in the field by Spada [1], Fernandes [2] and Costa [4], are presented respectively in Tables 2, 3 and 4. The parameters of each layer were defined based on properties of the materials reported by authors.

Table 2 Comparison of deflections measured in the field by Spada, [1]

Track section	Track modulus [MPa]	Deflection				Real properties within the simulation range?
		Field measure [mm]	Estimated by Eq. (3) [mm]	With standard deviation as error (± 23.35 %)		
				Lower limit	Upper limit	
C	47.25	2.68	3.16	2.42	3.90	No
OC	41.15	3.46	3.42	2.62	4.22	No
BR	63.98	2.82	2.69	2.06	3.32	No
R	43.4	4.24	3.31	2.54	4.08	No
NI	54.1	8.12	2.93	2.24	3.61	No
EP	63.60	2.15	2.69	2.06	3.32	No
DC	54.00	5.48	3.00	2.30	3.70	No

Table 3 Comparison of deflections measured in the field by Fernandes, [2]

Track section	Estimated track modulus [MPa]	Deflection					Real properties within the simulation range?
		Field measure [mm]	Estimated by Eq. 3 [mm]	Estimated by rectified equation [mm]	With standard deviation as error (± 23.35 %)		
					Lower limit	Upper limit	
01	39.93	1.32	3.09	1.19	0.91	1.47	No
02	39.93	1.32	3.09	1.19	0.92	1.47	No
03	64.61	0.92	2.36	0.46	0.35	0.57	No
04	25.64	1.84	3.96	2.06	1.58	2.54	No
05	54.87	1.04	2.59	0.69	0.53	0.85	No
06	8.87	4.08	7.09	5.19	3.98	6.40	No
07	8.99	4.04	7.04	5.14	3.94	6.34	No

Table 4 Comparison of deflections measured in the field by Costa, [4]

Ballast condition	Load [kN/axle]	Sleeper type	Track modulus calculated by Costa [4] [MPa]	Deflection					Real properties within the simulation range?
				Field measure [mm]	Estimated by Eq. 3 [mm]	Estimated by rectified equation [mm]	With standard deviation as error (± 23.35 %)		
							Lower limit	Upper limit	
New with 30 cm thickness	58	Concrete	42	0.37	1.37	0.53	0.40	0.65	No
	281		50	1.57	3.39	1.49	1.14	1.84	Yes
	93		34	0.63	2.15	0.25	0.19	0.31	No
	316		84	1.19	2.73	0.83	0.64	1.03	Yes
Unfurnished with 40 cm thickness	93*	Concrete	5	3.05	6.59	4.69	3.60	5.79	No
	316		20	3.45	5.98	4.08	3.13	5.03	Yes
	318		49	1.80	3.69	1.79	1.37	2.20	Yes
	318		43	1.97	3.96	2.06	1.58	2.54	Yes
Clogged with 27 cm thickness	80	Wood	6	2.10	5.43	3.53	2.70	4.35	No
	316		20	3.50	5.97	4.07	3.12	5.02	Yes
	50		9	1.02	3.23	1.33	1.02	1.64	No
	316		28	2.73	4.98	3.08	2.36	3.80	Yes
	318		17	3.92	6.55	4.65	3.56	5.73	Yes

* *Costa [4] believes that the higher deflection measured for a load of 93 kN/axle compared to 316 kN/axle is due to the gap between sleeper and ballast*

4 Conclusions

The deflections estimated were satisfactorily compared to the values measured in the field by Spada [1], Fernandes [2] and Costa [4] in several railroads within the Brazilian territory, especially in tracks with parameters within the simulated limits. Based on parametric analysis and the literature, is concluded that:

a) 40 % of the simulations resulted in deflections up to 2 mm, 74 % up to 3 mm and 91 % up to 4 mm. Since only tracks in perfect condition were simulated, it can be stated that 4 mm would be a suitable maximum value so that a good railway track is guaranteed;
b) 9 % of the track modules were between 10 and 20 MPa, 18 % between 20 and 30 MPa, 22 % between 30 and 40 MPa, 26 % between 40 and 50 MPa, 16 % between 50 and 60 MPa, 6 % between 60 and 70 MPa, 3 % above 70 MPa.
c) The load is the main responsible for the magnitude of the deflection, all other track properties have a secondary effect;
d) The rail has the important function of distributing the load to the sleepers and is the layer with the highest relevance to fight against excessive deflections. The stiffer the rail, better will be the stress distribution and smaller the deflections tend to be;
e) The greatest deflections occur in tracks with steel sleepers, followed by wooden sleeper and concrete monoblock sleepers;
f) When the deterioration of the ballast layer is not considered, this layer does not significantly influence deflection. However, the literature reports that is one of the layers which greater influences in permanent deformation over time and thus, it is necessary to use a model that takes this into consideration in case of degradation analysis, as proposed by Indraratna et al. [16];
g) The grained sub-ballast was not relevant for the measured deflections. Its contribution is more related to particle size transition between the ballast and the subgrade, being a layer of importance to guarantee track durability, avoiding the acceleration of ballast degradation;
h) The subgrade is an mportant layer in controlling deflection, especially when the upper layers do not help for stress distribution. Stiffer subgrades result in smaller deflections, while less stiff subgrades result in greater deflections. The layer's resilient modulus is closely related to humidity, again underscoring the sub-ballast importance in track durability.

Not necessarily pavements with smaller track modulus have higher deflections and inferior qualities, since similar pavements can result in different track modulus, for example, changing only the sleeper. Therefore, the track modulus should not be seen as a qualitative parameter of the railway track, affirmation found in several literatures, but as a representative parameter of the overall behavior of the pavement according to the properties of the layers that compose it. The track modulus is an identity for a particular track and quality should be analyzed based on excessive deflections, ballast degradation, rate of insufferable sleepers, and wear of rails and not just considering the track modulus. In fact, the greater is the track modulus the greater is the rigidity of the railway track, after all it is a parameter of rigidity, however a high rigidity cannot always be coupled to a satisfactory condition of the track.

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