CETRA2020* 20-21 May 2021, Zagreb, Croatia
6th International Conferenceon Road and Rail Infrastructure

TRACK BED DESIGN AND EVALUATION METHODS

Otto Plášek, Petr Frantík, Nikola Svobodová
Brno University of Technology, Faculty of Civil Engineering, Czech Republic

Abstract

Track bed consists of structural layers and subsoil, and in the case of the ballasted track, also the structure comprises ballast bed. Together, this structure creates a layered half-space in which the load from the rail supports is distributed. The paper is focused on static analysis of track bed as a layered half-space structure.
Currently, the so-called DORNII method, developed in the Soviet Union in the mid-1950s, is still used for the assessment of the track bed structure. The evaluation of this structure is based on the assessment of the deformation resistance. The deformation resistance is assessed through the value of the deformation modulus of the sub-ballast surface and the subsoil surface. The deformation modulus is measured both in the geotechnical survey and in the acceptance process of construction works. When designing the track bed, the deformation modulus is taken into account as an essential material characteristic.
Neither the stress values relative to the depth nor deflection is analysed in the currently used methodology. The DORNII method is empirical but allows calculation of vertical stress and deflection. The authors wondered whether this method would not be too inaccurate because of increasing train speed and axle load. Two methods, analytical and finite element, were chosen for comparison. The article describes the specific analysis procedures and compares their results.

Keywords: railway substructure, track bed, static analysis, substructure design, stress diagnostic

1 Introduction

At present, the trend of increasing the design speed of railway lines is quite evident, and in the case of new lines, the construction of high-speed lines prevails. As the speed of rolling stock increases the dynamic load on substructure increases. Therefore, it is necessary to pay attention to the entire track bed's sufficient deformation resistance during the design. This deformation resistance should guarantee support for the track structure, so that there are no significant, and especially not permanent, deflections under passing rail vehicles. Irreversible deflection of the track bed causes a permanent change in the track quality and thus, in turn, greater dynamic load on the railway superstructure and substructure, which, among other things, results in a significant reduction in the lifetime of the track [1].
Insufficient deformation resistance, therefore, leads to track undesirable settlement. To limit these settlements, it is necessary to well identify the processes that take place in the track bed due to loading. This is relatively difficult because the space under the track skeleton is not limited. Several different empirical and numerical methods attempt to describe these mechanical processes.

DOI: https://doi.org/10.5592/CO/CETRA.2020.1145

One of the mentioned methods is the DORNII (Doržnyj naučno-issledovateľskij institut) method [2]. It is a partly analytical and partly empirical method. It is included in the Czech infrastructure manager regulation and serves to design structural layers of the railway substructure concerning deformation resistance [3]. With its use, it is possible to determine the fields of stresses and displacements in the track. The question remains whether this is a realistic view and a suitable tool for designing a deformation-resistant structure, as this method simplifies and omits many things. The semi-analytical method of layered half-space (LHSM), used in road engineering, will be used for its comparison. The second one is the finite element method (FEM), a numerical method for solving differential equations with a relatively wide scope. Both of these methods provide a comprehensive overview of stresses and displacements in the substructure.

At the end of the article, the results of the calculation of displacements and stresses calculated according to DORNII, LHSM and FEM for a typical structural composition of the railway substructure in poor geotechnical conditions are tested.

2 Description of analysis methods

2.1 DORNII Method

Currently, in the Czech Republic, the DORNII method is used to determine the deformation resistance. The method is partly empirical and partly analytical. N. N. Ivanov developed it in the 1950s [2]. The method allows calculating the equivalent deformation modulus of a multi-layered track bed structure. In particular, such thicknesses or modulus of deformation of the track bed layers are sought so that the resulting modulus of deformation E_{eq} of the whole system guarantees the required deformation resistance.

In order to determine the required equivalent value of deformation of the whole structure E_{eq}, it is necessary to know the deformation characteristics of the materials used for individual layers (E_i, v_i). The DORNII method considers soil deformation only in the column of material that is loaded. This fact means that neither shear stress nor horizontal stress in the soil, which would otherwise arise at the two structural layers' interface, is considered. Based on the measurement of stress on the ground plane, M. I. Jakunin [2] recommended the following empirical relationship for the course of stress to a depth of 1.0 m:

$$\sigma_z = \frac{q}{1+\eta\left(\frac{z_e}{D}\right)^2} \tag{1}$$

in which: q is the pressure from the vehicle wheel acting on the ground surface [MPa], z_e is the design depth a quasi-homogeneous space [m], D is the diameter of the reference circular area of the wheel pressure [m], *n* is a dimensionless parameter, in a two-layer system that *n* = 1, in a single layer *n* = 2.5 (when $z_e = z$).

The equivalent depth z_e is determined according to the equation (see Fig. 1):

$$z_e = z + h_e - h_1 = z + (n-1)\cdot h_1 \tag{2}$$

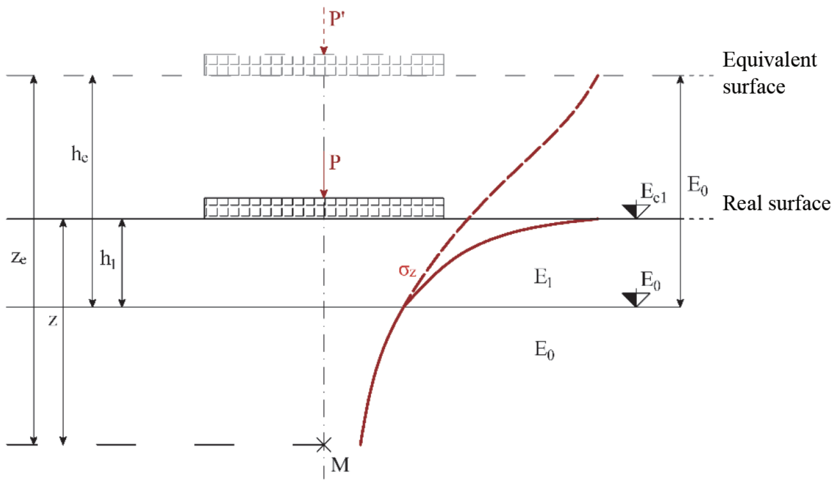

Figure 1 Pokrokovský's model [3] of equivalent layer

The equivalent deformation modulus and vertical deflection for a two-layer structure can be expressed:

$$E_{e1} = \frac{E_0}{n^{2,5}\left[1 - \frac{2}{\pi}\left(1 - \frac{1}{n^{3,5}}\right).arctg\left(k_2.n\right)\right]}; \quad n^{2,5} = \frac{E_1}{E_0} \tag{3}$$

$$y_1 = \frac{p \cdot D}{E_0}\left[\frac{\pi}{2} - \left(1 - \frac{1}{n^{3,5}}\right) \cdot arctg\,\frac{h_1 \cdot n}{D}\right] \tag{4}$$

where: $n^{2,5}$ is the deformation characteristic of the system, E_0 is the deformation modulus of the lower layer material [MPa], E_1 is the deformation modulus of the upper layer material [MPa], E_{e1} is the equivalent deformation modulus of the whole structure [MPa], h_1 is the upper layer thickness [m], D is the diameter of the circular load plate [m].

2.2 Layered half-space method

The layered half-space method was initially developed for road engineering [4], [5]. Describes a situation where a vertical circular wheel load is applied to the surface of a multilayer structure. The circular load is idealized because other load components act on the road. In addition to this vertical load, which comes from the vehicle's weight, there is a horizontal surface load due to acceleration and braking and a shear load due to centrifugal forces in the directional curves.

In railway structures, the distribution of forces from railway vehicles' running occurs in a completely different way. However, the layers' design concerning the railway substructure's deformation resistance is based on applying pressure using a circular plate to the structural layer. The same task can be investigated by the layered half-space method.

The spatial problem of the pressure acting from a circular load plate of diameter D on the track bed's structural layer is transformed into a two-dimensional problem in the layered half-space method since it is an axially symmetric problem. That means with a suitable location of the origin of the reference axes in the centre of the loaded circular surface, the task is transformed from Cartesian coordinates to cylindrical coordinates, and thus axial symmetry and significant simplification are achieved.

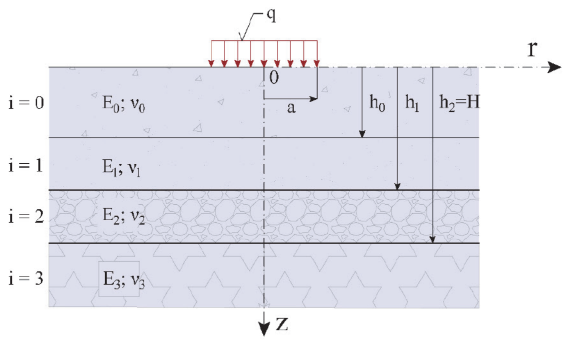

Figure 2 Layered half-space scheme

The calculation of the stress and displacement field is based on the solution of the compatibility condition, which is expressed by the following equation:

$$\nabla^4 \phi = 0 \tag{5}$$

in which ϕ is the stress function. This stress is considered for each of the layers. For a system with axially symmetric stress distribution, the differential operator ∇^4:

$$\nabla^4 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \tag{6}$$

where r is the coordinate for the radial direction and z for the vertical direction. The initial equation (4) is a fourth-order differential equation, so the resulting equations describing stresses and displacements will contain four integration constants, which can be calculated from boundary conditions and compatibility conditions. It is assumed:

$$\rho = \frac{r}{H}; \qquad \lambda = z / H \tag{7}$$

where r is radial coordinate, H is the total thickness of the structure, and z is the defined depth. The stress function given by the Eq. (4) is:

$$\phi_i = \frac{H^3 J_0(m\rho)}{m^2}\left[A_i e^{-m(\lambda_i - \lambda)} - B_i e^{-m(\lambda - \lambda_{i-1})} + C_i m\lambda e^{-m(\lambda_i - \lambda)} - D_i m\lambda e^{-(\lambda - \lambda_{i-1})} \right] \tag{8}$$

Function φ_i is the stress function for the i-th layer that satisfies the initial equation and in which J_0 represents a Bessel function of the first kind and zero-order; m is a parameter, A_i, Bi, C_i; D_i are integration constants that are determined from boundary conditions and compatibility conditions. The index i takes values from 1 to n and refers to the layer's order, starting with the surface layer. Substituting this equation into the Eq. (7), we obtain calculation equations for the quantities of normal stress, shear stress, vertical and radial displacement, generally R*.
We use the Hankel transform to find stresses and displacements for a uniform load q distributed on a circular surface, defined by radius:

$$\bar{f}(m) = \int_0^{\alpha} q\rho J_0(m\rho)\, d\rho = \frac{q\alpha}{m} J_1(m\alpha) \tag{9}$$

where α = a / H. Hankel's inverse transform $\bar{f}$ (m) is:

$$q(\rho) = \int_0^{\infty} \overline{f}(m) m J_0(m\rho) dm = q\alpha \int_0^{\infty} J_0(m\rho) J_1(m\alpha) dm \tag{10}$$

Assuming the load q negative, the corresponding quantity R is calculated:

$$R = q\alpha \int_0^{\infty} \frac{R^*}{m} J_1(m\alpha) dm \tag{11}$$

The layered half-space solution assumes that on all layers interface the same normal stress, shear stress, vertical displacement, and radial displacement occur. To calculate the 4n constant A_i, Bi, C_i; D_i boundary conditions are introduced:

$$\left(\sigma_z^*\right)_i = \left(\sigma_z^*\right)_{i+1}; \left(\tau_{rz}^*\right)_i = \left(\tau_{rz}^*\right)_{i+1}; \left(w^*\right)_i = \left(w^*\right)_{i+1}; \left(u^*\right)_i = \left(u^*\right)_{i+1} \tag{12}$$

On a surface for which i = 1 and $\lambda = 0$, the following boundary conditions apply:

$$\left(\sigma_z^*\right)_1 = -mJ_0(m\rho); \left(\tau_{rz}^*\right)_i = 0 \tag{13}$$

The stresses and displacements must logically disappear together with the increasing depth ($\lambda \to \infty$), therefore for the lowest layer i = n the following applies:

$$A_n = C_n = 0 \tag{14}$$

The solution was refined for the surface using Richardson's extrapolation, where the integral in Eq. (10) [6]:

$$I = \int_0^{\infty} f(\xi) d\xi \tag{15}$$

was modified to:

$$I = \lim_{b \to \infty} \int_0^{\infty} f(\xi) e^{-\xi^2 b} d\xi \tag{16}$$

2.3 Finite element method

The model for FEM analysis was created in the PLAXIS 3D program. The dimensions of the rectangular model in this analysis were set to dissipate the stress in its peripheral parts. The stress components expressing the stress and displacements did not change. The model's specific dimensions were set to 6 m in width and length and 7 m in depth. The load was placed in the centre of the model.

3 Example of track bed design

The particular analysis methods were compared for several specific track bed structures. Below is a specific case of such an analysis.

3.1 Input parameters

A uniform load q = 0.2 MPa was applied at the level of the bottom of the railway body to an area with a diameter of the circular plate D = 0.30 m. The deformation modules at the substructure plane level and the subsoil plane were determined to respect the required minimum deformation resistance, which must match regulation rules for the reconstructed line for speeds of 120 km.h^{-1} to 160 km.h^{-1}.

Oedometric modulus and Poisson's ratio were chosen as input deformation characteristics for the DORNII method. The oedometric modulus was selected for calculation because horizontal deformations are not considered when loading with a circular plate. Permanent deformations are included in the analysis the same way, as is the case with the oedometric test. The modulus of elasticity and the Poisson's ratio were used for the layered half-space method. A constitutive relation that links tension and transformation is Hooke's law for the LHSM. The modulus of deformation and the Poisson's ratio were chosen for the finite element method. A linearly elastic model was used as a constitutive relation.

3.2 Example of track bed analyses

Track bed with a layer of stabilized soil is specially designed when necessary to increase the subsoil's strength and deformation resistance, further if required reduction of the thickness of the structural (base) layer or improve the subsoil resistance to frost.

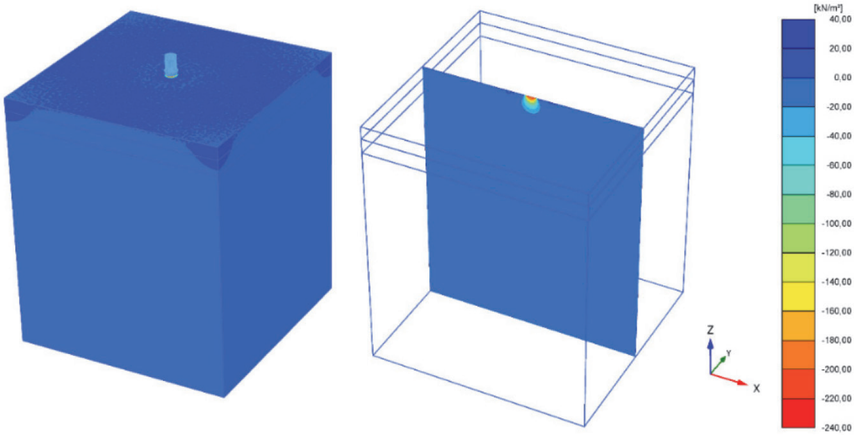

Figure 3 Vertical stress σ_z [kPa] calculated by FEM in PLAXIS software

Layers of following input parameters were considered: Fine crushed stone mixture 0/32, E_{def} = 80 MPa, E_{oed} = 96 MPa, E = 89.6 MPa, v = 0.25, h = 0.35 m; Lime soil stabilization, E_{def} = 70 MPa, E_{oed} = 84 MPa, E = 78.4 MPa, v = 0.25, h = 0.4 m; Clay with high plasticity, E_{def} = 8 MPa, E_{oed} = 17.1 MPa, E = 13.9 MPa, v = 0.4.

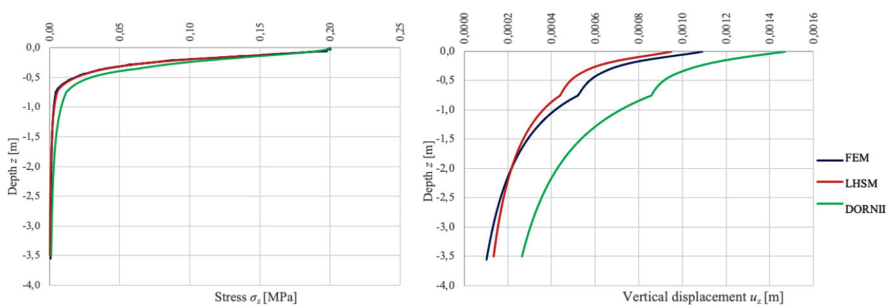

Figure 4 Comparison of vertical stress and vertical displacement under the centre of applied load for methods used

From the comparison of the results, it can be observed that there are no significant differences between the methods for stress results. The course of stress along the depth is not so smooth, which is given by the relative flexibility of the last layer; however, all three methods treat this fact in the same way. The stresses, determined by the methods of FEM, LHSM and DORNII, do not differ significantly in principle. Besides, stress is a theoretical quantity that only describes the state of the structure. The link between stress and strain is Hooke's law, so it is impossible to conclude purely based on knowledge of stress whether the strain resistance of a system, assessed by any method, is sufficient. The vertical deflection better expresses the deformation resistance. The more flexible the structure, the greater the deflections will occur. In this, DORNII was very consistent, and the largest decrease was always calculated for all types of structures compared to FEM and LHSM. If we wanted to limit the vertical deflection and get, for example, to the value found by the LHSM, which was the lowest for all cases, we would have to design based on DORNII a structure considerable rigid.

4 Conclusions

So DORNII does not correctly describe the processes that take place in the substructure. It neglects horizontal deformations, but as a tool for designing a sufficiently deformation-resistant railway substructure, it will stand on the safe side. The calculated equivalent modulus of the system, based on this theory, is very likely to be lower than the real one. So DORNII works on the safe side. In addition, it is incomparably computationally simpler.

Trends in the calculated results of stresses and displacement received by particular methods were compared up to a depth of 3.5 m. The stresses along depth did not differ almost for all three methods. The resulting deflection observed in the FEM was always greater than in the LHSM, however not significantly. The results of vertical displacement were always almost the same for both of these purely computational methods. In contrast, the total deflection at DORNII was up to a third larger for all investigated track bed structures compared to the LHSM calculations.

The disadvantage of the DORNII method is its limitations concerning the track bed structure. The method requires that the deformation modulus of the layers grow upwards in the structure; furthermore, it is not possible to include any geosynthetic reinforcement elements in the calculation. The DORNII method does not provide stress and displacement fields in the results and does not allow assessing the track bed structure using the limit state method. Since it does not provide a reliable calculation of the vertical deflection, it cannot be used to design the track elasticity.

The DORNII method is very simplified; for example, it omits horizontal strain and stresses. On the other hand, there are very good experiences with FEM in geotechnics, it provides very high-quality outputs and describes mechanical processes more comprehensively. Its results were very close to the layered half-space method. Most likely, DORNII does not have sufficient informative value about what actually happens in the substructure and does not describe these processes' consequences (too high deflection). However, as a methodology for designing a track bed structure, it guarantees greater deformation resistance than the case with FEM and Layered half-space method.

Acknowledgment

This paper has been worked out under the project No. FV40081 "Advanced technologies for the construction and renewal of structural layers of the railway substructure with the effective use of materials from secondary raw materials", supported by Ministry of Industry and Trade.

References

[1] Lichtberger, B.: Track compendium. 2. vyd. Hamburg: DVV Media Group, 2011. ISBN 978-3-7771-0421-8.

[2] Babkov, V.F., Bykovskij, N.I., Gerburt-Gejbovič, A.J.: Gruntovedenie i mechanika gruntov, Moscow, Dorizdat, 1950.

[3] Ižvolt, L.: Železničný spodok: namáhanie, diagnostika, navrhovanie a realizácia konštrukčných vrstiev telesa železničného spodku, Žilina, Žilinská univerzita v Žilině, 2008.

[4] Huang, Y.H.: Pavement Analysis and Design, Prentic–Hall. 2004.

[5] Maina, J.W., Matsui, K.: Developing software for elastic analysis of pavement structure responses to vertical and horizontal surface loadings, Transportation Research Record, 2004, pp. 107–118, doi:10.3141/1896-11

[6] Maina, J., Kunihito, M.: Elastic multi-layered analysis using DE-integration. Publications of the Research Institute for Mathematical Sciences, 41 (2005) 4, pp. 853–867