STUDY OF THE RAILWAY CURVES GEOMETRY UNDER THE EFFECT OF RAIL TRAFFIC OVER TIME

Zsolt László Orbán[1], Marius Gocica[1], Laurenţiu Mărculescu[2]
[1] *Technical University of Cluj-Napoca, Romania*
[2] *Baicons Impex S.R.L., Romania*

Abstract

Over time, under the action of moving railway vehicles and temperature variations, the railway track deforms both in alignment and especially in the areas of the curves, the degradation process being self-generating. Given that the rehabilitation works of the Romanian railways are difficult, monitoring the geometry of the track and carrying out the necessary maintenance works in order to restore the geometric elements, respectively improving them for maintain traffic speeds are the main concerns of the Romanian Railway Administration. This paper studies the variation over time (years) of the geometry of some railway curves under the action of railway traffic and highlights the transformations that these curves have gone through, with emphasis on changing the geometric characteristics of the transition curves following the maintenance works caried out on these railway sections. Although the possibilities for improving existing situations on the field have been limited, efforts have been made to find optimal ways to redesign these curves. The paper presents the medium-term life cycle, 10-15 years, of some railway curves and proposes some ways of optimizing the transition between the alignments and the circular curves on the studied areas.

Keywords: railway curve, transition curve, retracking, angular diagram

1 Introduction

Monitoring the geometry of the track and performing the necessary maintenance works in order to restore the geometric elements in plan, respectively improving them in order to be able to maintain optimal traffic speeds, is the main concern of the Romanian Railway Administration.

This paper studies the variation in time, between 2008 and 2021, of the geometry of the railway curves under the action of rail traffic and highlights the transformations that these curves have "undergone", with emphasis on changing the geometric characteristics of the transition curves, following maintenance work that intervened on their geometry.

Several isolated curves were studied on the railway line "Magistrala 300" that connects Câmpia Turzii and Cluj-Napoca.

Simple curves were chosen, single-radius curves, and the connection of the circular curves with the alignments preceding and succeeding the curves in all cases were cubic parabolas.

DOI: https://doi.org/10.5592/CO/CETRA.2022.1459

2 Transition curves

2.1 The conditions that a progressive transition curve must meet

A connecting curve between an alignment and a circular curve, or between two successive circular curves with different radii, must meet several conditions in order to perform:
• to achieve a continuous and uniform variation of the centrifugal force;
• to achieve a continuous and uniform variation of the superelevation (and widening) of the track;
• the above variations should not exceed certain limit values.

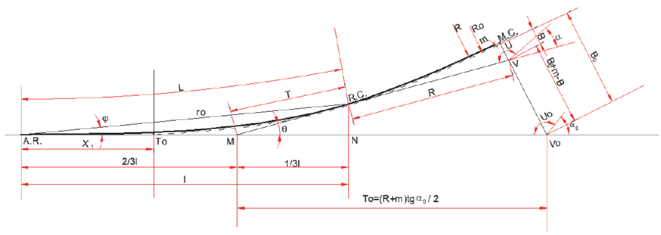

Figure 1 The elements of a progressive transition curve

Considering the transition curve defined by a function y = f (x), the above conditions are translated more concretely by the following:

1. The variation of the transition curve should be continuous and uniform along its entire length. The function f (x) must therefore be continuous and monotone.
2. At point AR the transition curve must have a common tangent to the alignment, and at point RC the transition curve will have a common tangent to the arc of the circle. The tangent will have a continuous and uniform variation along the entire length of the transition curve. Therefore, the function f'(x) (the first derivative) must be continuous and monotonous.
3. The curvature will have the 0 value at point AR and the value 1/R at point RC. The tangent will have a continuous and uniform variation along the entire length of the transition curve. Therefore the function f"(x) (the second derivative) must be continuous and monotonous.
4. The tangent of the superelevation curve must be zero at points AR and RC, i.e., at these points the wheels on the outer wire must not produce vertical shocks. The tangent of the transition curve represented in the vertical plane must have a continuous and monotonous variation on the length of the transition curve. In most cases the superelevation is introduced along the length of the connection curve and is proportional to the value of the normal acceleration, i.e., proportional to the value of the curvature. The elevation and curvature will have the same law of variation.
5. The latter condition should be met in case of higher degree curves, used on high-speed lines. The lifting acceleration of the wheel on the outer wire should not occur or disappear suddenly along the length of the transition curve. The acceleration will be zero at points AR and RC and will vary continuously and monotonously along the length of the transition.

In order to perform well, a transition curve must meet, in addition to the condition of providing comfort, other conditions as well:
• to ensure high speed traffic;
• not to have an exaggerated length;

• not to cause heavy wear of the rails or wheels;
• to be easily built in the track;
• easy to maintain.

These additional conditions impose certain limitations on the cantilever transition curve:
1. the maximum ramp of the superelevation curve must not exceed a limit value;
2. the speed at which the wheel of the vehicle climbs the ramp must not exceed a certain limit value;
3. the acceleration of the superelevation variation must not exceed a certain limit;
4. the variation over time of the uncompensated superelevation must not exceed a certain limit value;
5. the acceleration of the variation of the uncompensated superelevation must not exceed a certain limit value;
6. the kinetic energy lost when passing from the alignment to the curve must not exceed an allowable limit value;
7. the transition curve must be capable of being traced and maintained;
8. the permissible tracing error must not be exceeded;
9. to be able to describe the dependence between superelevation and versine; the superelevation being proportional to the curvature, it is also proportional to the versine;
10. the original curve must retain all its properties following homothetic transformations.

2.2 Studied progressive transition curves

A large number of algebraic or trigonometric functions can be found to satisfy the conditions listed above. At the time of designing the studied curves, the Romanian Railway Administration used a progressive transition curve that would correspond best to an optimal traffic and at the same time not be difficult to trace or maintain for that period of time. This progressive curve is the cubic parabola, which is actually a feature of the clothoid by considering the first terms of the parametric equations that define the clothoid.
Other progressive curves were considered in the study. In the following we will briefly present the studied progressive curves.

2.2.1 The clothoid (Euler spiral)

The parametric equations that define the clothoid are:

$$x = l_x - \frac{l_x^5}{40 \cdot R^2 \cdot L^2} + \frac{l_x^9}{3456 \cdot R^4 \cdot L^4} - \frac{l_x^{13}}{599040 \cdot R^6 \cdot L^6} + \dots \tag{1}$$

$$y = \frac{l_x^3}{6 \cdot R \cdot L} - \frac{l_x^7}{336 \cdot R^3 \cdot L^3} + \frac{l_x^{11}}{42240 \cdot R^5 \cdot L^5} - \frac{l_x^{15}}{9676800 \cdot R^7 \cdot L^7} + \dots \tag{2}$$

Where:
L – the total length of the progressive curve;
l_x – the length of the progressive curve from its beginning to a current point on it;
R – radius of the circular curve.

The angle formed by the tangent to the progressive curve at a current point with the extension of the alignment is:

$$\varphi_x = \frac{l_x^2}{2 \cdot R \cdot L} \tag{3}$$

2.2.2 The cubic parabola

The use of the exact calculus relations of the clothoid is difficult, so practically the calculations are made taking into account only the first terms of the parametric equations, thus obtaining the relations that define the cubic parabola:

$$x \cong l_x \tag{4}$$

$$y = \frac{l_x^3}{6 \cdot R \cdot L} \tag{5}$$

The cubic parabola is connected with the circular curve at the RC point (fig. 1). At this point there is always an ordinate non-closure ($y_C \neq y_P$) due to the fact that in the parametric equations of the cubic parabola only the first terms of the development of the clothoid parametric equations were taken into account. In order to perfectly connect the cubic parabola with the circular curve, a correction is made to the equation of the cubic parabola, the new parametric equation being called the improved cubic parabola

$$x \cong l_x \tag{6}$$

$$y = \left[1 + \left(\frac{L}{2 \cdot R}\right)^2\right]^{\frac{3}{2}} \cdot \frac{l_x^3}{6 \cdot R \cdot L} \tag{7}$$

2.2.3 The 4th degree parabola

The parametric equations that define the 4th degree parabola are determined separately on each half of the progressive curve. The simplified equations of the 4th degree parabola are presented.
For the first half of the progressive curve ($0 \leq l_1 \leq L/2$)

$$\varphi_{1x} = \frac{2 \cdot l_{1x}^3}{3 \cdot R \cdot L^2} \tag{8}$$

$$x_1 = l_{1x} \tag{9}$$

$$y_1 = \frac{l_{1x}^4}{6 \cdot R \cdot L^2} \tag{10}$$

For the second half of the transition, the origin of the coordinate system is transferred to the last point of the transition from the first half ($0 \leq l_1 \leq L/2$):

$$\varphi_{2x} = \frac{L}{12 \cdot R} + \frac{l_2}{2 \cdot R} + \frac{l_2^2}{R \cdot L} - \frac{2 \cdot l_{2x}^3}{3 \cdot R \cdot L^2} \tag{11}$$

$$x_2 = l_{2x} \tag{12}$$

$$y_2 = \frac{L}{12 \cdot R} \cdot l_{2x} + \frac{l_{2x}^2}{4 \cdot R} + \frac{l_{2x}^3}{3 \cdot R \cdot L} + \frac{l_{2x}^4}{6 \cdot R \cdot L^2} \tag{13}$$

2.2.4 The sinusoidal progressive curve

The parametric equations of the sinusoidal progressive curve are:

$$\varphi_x = \frac{1}{R} \cdot \left(\frac{l_x^2}{2 \cdot L} - \frac{L}{4 \cdot \pi^2} + \frac{L}{4 \cdot \pi^2} \cdot cos \frac{2 \cdot \pi \cdot l_x}{L} \right) \tag{14}$$

$$x \cong l_x \tag{15}$$

$$y = \frac{l_x^3}{6 \cdot R \cdot L} - \frac{L}{4 \cdot R \cdot \pi^2} \cdot \left(l_x - \frac{L}{2 \cdot \pi} \cdot sin \frac{2 \cdot \pi \cdot l_x}{L} \right) \tag{16}$$

2.2.5 The cosine progressive curve

The parametric equations of the cosine progressive curve are:

$$\varphi_x = \frac{1}{2 \cdot R} \cdot \left(l_x - \frac{L}{\pi} \cdot sin \frac{\pi \cdot l_x}{L} \right) \tag{17}$$

$$x \cong l_x \tag{18}$$

$$y = \frac{l_x^2}{4 \cdot R} - \frac{L^2}{2 \cdot R \cdot \pi^2} \cdot \left(1 - cos \frac{\pi \cdot l_x}{L} \right) \tag{19}$$

In Figure 2 are represented, for a given length L of the progressive curve, the five types of progressive curves (cubic parabola, improved cubic parabola, 4th degree parabola, sinusoidal and cosine progressive curve).

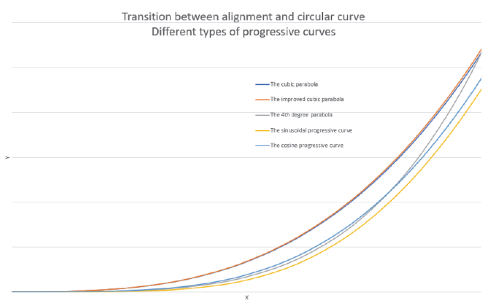

Figure 2 Comparative graphic for different progressive curves

3 Investigating methods for the studied curves - Principles

The method used to study these railway curves was defined by the data found in the measurement records over the 12-13 years. Because these data refer to measured values of versine and superelevation, the method chosen for the study was that of the angular diagram.

An optimal passing in a curve (C) or group of curves means a corresponding variation of the curvature and therefore a continuous variation of the versine at each point of the curve. If the curvature variation law of the (C) curve is known:

$$f_c(l) = 1/\rho \tag{20}$$

and knowing that:

$$d\varphi = dl/\rho \tag{21}$$

then:

$$d\varphi = f_c(l)\, dl \tag{22}$$

respectively:

$$\varphi = \int f_c(l)\, dl = f_\varphi(l) \tag{23}$$

Knowing the function that defines the variation of the angle f, it can be represented graphically in a system of axes f0l, obtaining the angular diagram for the curve (C). Therefore, if the law of variation of the curve is known, the angular diagram for any curve can be drawn.

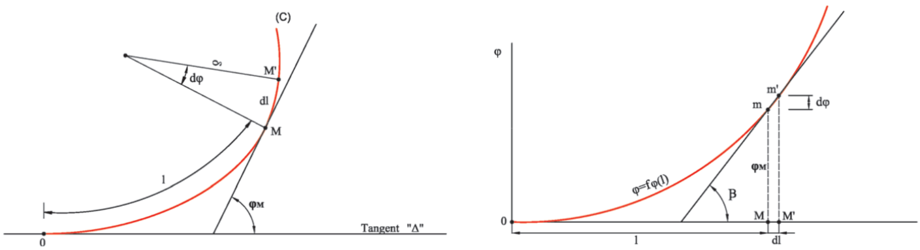

Figure 3 The curve in plane representation and the angular diagram

If drawing the angular diagram for an undeformed curve for which we know the curvature variation law should not be a problem, for a deformed curve (with an irregular shape), the angular diagram cannot be determined according to those presented above. In this case, an approximate method (Nalenz procedure) is used, which involves determining the elements defining the angular diagram from known data such as versine (f_i) measured in the middle of a string of known length (c) at dividing points materialized in the field at constant distances (Dl). To facilitate the calculation, it is approximated that the length of the string and of the curve arc between two division points are equal and have the value c/2, respectively the curve between two consecutive division points is approximated by an arc of a circle.
In the figures below it can be seen the principle that determines the angle that the tangent makes at any point on the curve with the initial alignment based on the measured versine.

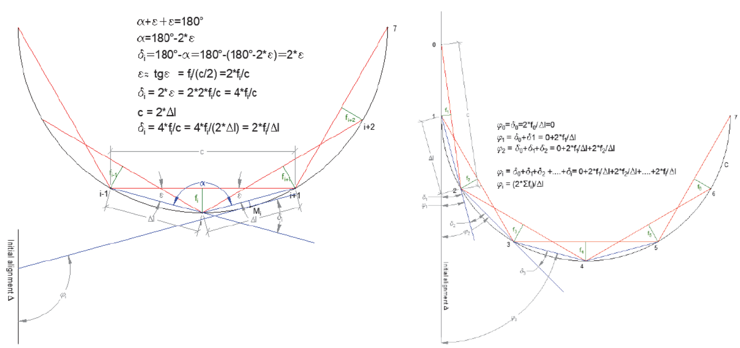

Figure 4 Determining the angles according to the versine

In conclusion, it can be stated that the ordinate in the angular diagram for the point studied on the curve at the middle of the distance between the division points (i) and (i+1) is obtained by summing the versine values up to the division point (i) and multiplying it by a constant value of 2/Dl.

$$\varphi_i = \frac{2}{\Delta l}\sum_{0}^{i} f_i \tag{24}$$

Obviously, it can be represented on the same graph an angular diagram for a deformed curve and a diagram for a designed curve. The differences between the ordinates of the same point on the abscissa, drawn at the same scale, represent the angular difference Df_i in the studied point. With these differences between the deformed and the projected diagram it is possible to find out the repair values in each considered point.

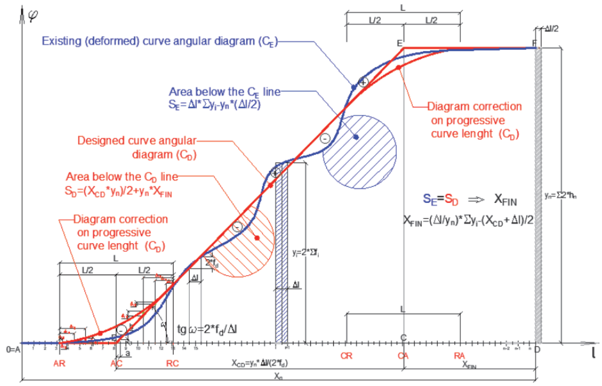

Figure 5 Deformed and designed curve angular diagram (theoretically)

The characteristic of this method is that it uses as a baseline the very initial curve (not retracted) on which the versine values are measured on the outer wire of the curve. Depending on the measurement base (distance between dividing points) these values were equivalent to a string length of 20 m, i.e., values of versine measured for every 10 m. These arrows are summed successively, the partial sums being processed and represented graphically obtaining the angular diagram for the deformed curves measured annually between 2008 and 2021. It has to be mentioned that the measurements were performed approximately in the same season of the year.

4 Case study

On the Romanian railway line ”Magistrala 300”, between the cities Câmpia-Turzii and Cluj-Napoca, railway track under the leadership of Cluj Regional Railway Administration, several curves were studied. The results and conclusions for two of these curves will be presented in this paper:

- Km 456+600, line 300, track 2, between Câmpia Turzii and Valea Florilor;
- Km 488+700, line 300, track 1, between Cojocna and Apahida.

The study in time of the geometry variation of these railway curves, due to the action of railway vehicles and maintenance works, was made by the angular diagram method based on data collected in the field (versine, superelevation) between 2008 and 2021. The calculations were made by a numerical application developed by the authors of this study. It should

be noted that in 2016 the curves on this section of the railway underwent to major interventions and were retracked, which will be seen in the represented annual diagrams.
The angular diagrams for the deformed curves are unique, based on the measurements of the versine values. The figures 6 and 7 below show the annual variations of these diagrams.

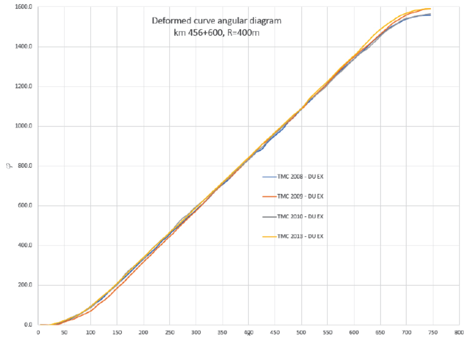

Figure 6 Deformed curve (km 456+600) angular diagram – period 2008-2013

There is an increasing tendency of the versine value in 2009 and 2013. It can also be seen that in 2008 and 2010 respectively the curve was rectified and a closer analysis shows that the lengths of the progressive curves - cubic parabolas - were shortened to around 60 m and the radius kept at 400 m. For the years 2009 and 2013, following the evolution of the versine values and applying a retraction variant for each case, there is a tendency to increase the length of the progressive curves to around 80 m to 90 m length.

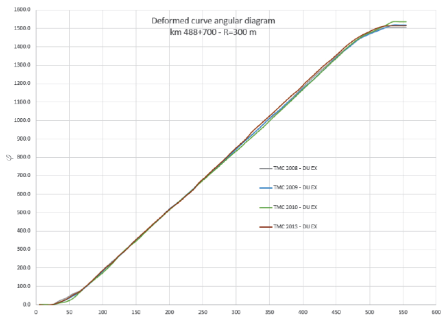

Figure 7 Deformed curve (km 488+700) angular diagram – period 2008-2013

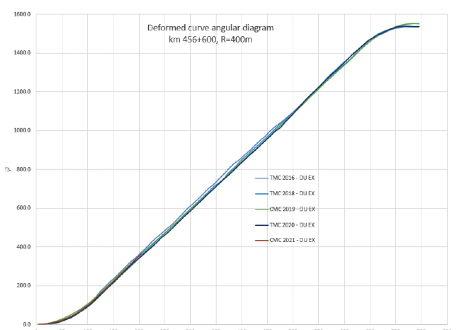

Figure 8 Deformed curve (km 456+600) angular diagram – period 2016-2021

It can be seen in the diagram above that after the capital intervention on the line on which the studied curve (km 456 + 600) is located, respectively its retraction, the variation of the annual value of the measured versine tends to be smaller than before the 2016 intervention.

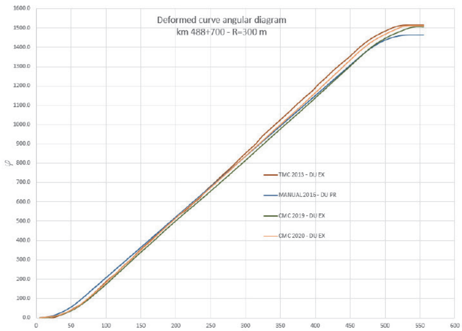

Figure 9 Comparison of angular diagrams for deformed curves (km 488 + 700) - period 2013-2020 with angular diagram for designed curve in 2016.

Analyzing the diagram in Figure 8, it can be observed that in 2016 this curve was retracted with a larger radius (318 m compared to the initial 300 m), shown by the fact that the angular diagram for the designed curve in 2016 has a smaller inclination than the angular diagram for the deformed curve in 2013. This new inclination seems to be maintained in the coming years (2019 and 2020), as shown by the diagrams for the deformed curves in these years.
Regarding the angular diagram for the designed curve, they were represented for each annual measurement and compared with the evolution of the curve geometry for the following years. The choice of the optimal designed curve must be made very carefully. Thus, in order to determine this line, which should correspond to the initial characteristics of the curve, the following requirements have been met:

- Maintain the initial deviation angle between the alignments adjacent to the curve by overlapping the end parallels of the E_C and C_D lines;
- The final retrack at the last division point should be null. This condition can be met if the C_D line is chosen in such a way that the sum of the areas between the E_C and C_D lines, respecting the sign convention, is zero;
- Choosing the C_D line so that it intersects the C_E line in as many points as possible in order to obtain the smallest possible repairs;
- As far as possible keep the inclination of the C_D line in such a way as to ensure a radius approximately equal to the known radius of the curve.

Taking in consideration the curve from km 456+600 and following the geometry evolution of this curve after 2016, several retraction variants were designed with the preservation of the initial deviation angle between the alignments, the choice of the inclination of the C_D line so as to keep a radius of 400 m. and ensuring zero final repair. These studied variants show a tendency to transform the geometry of the designed progressive curve, based on the measured versine values, to the type of cosine progressive curve.
In figure 10 below, it can be observed these diagrams, the first for 2016 before the intervention, and the next ones two respectively four years later.

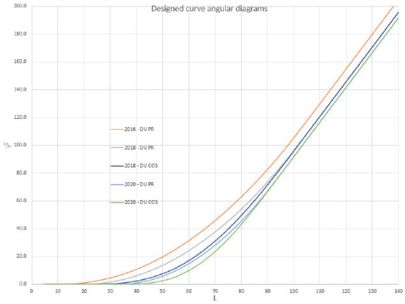

Figure 10 Angular diagrams evolution for designed curves (km 456 + 600) - period 2016-2020

As can be seen, special attention has been paid to the progressive curves that connect the circular curve with the alignments that precede and succeed the curve. In the study, the angular diagrams for the designed curve were represented, taking into account several types of progressive curves. As expected, the diagram of the designed progressive curve that "fitted" best on the distorted curve diagram was the cubic parabola. However, analyzing the angular diagram for the deformed curves, it was observed that the length of these cubic parabolas was significantly longer than the initial designed length, respectively the normal length provided in the instructions. On the other hand, if we analyze the angular diagram for the designed curves of this period of time, we can see that the shape of the progressive curves tends to be that of a cosine progressive curve. Since all the progressive curves studied are cubic parabolas, which satisfy approximately only three of the conditions mentioned in the paragraph 2.1., its scope is established by limiting the errors due to the neglect of the terms from the variation law of the clothoid, namely by:

- Limiting the sudden increase of ordinate $Dy <= Dy_{adm}$;
- Limiting the curvature difference $DC <= DC_{adm}$;
- Limiting the angular displacement $Dr <= Dr_{adm}$;

Thus, we can say that the maximum length of the progressive curve, in our case the cubic parabola, can be calculated from the relation:

$$L \leq \sqrt[4]{R^3} \tag{25}$$

If this condition is not met, an optimized version of the cubic parabola must be adopted, which in our literature is called the improved cubic parabola whose parametric equations have been highlighted at point 2.2.2 of this paper.

5 Conclusions

Analyzing the data collected in the field (versine), it can be seen that the length of the progressive curve (cubic parabola) applied in the field is greater than the value calculated according to the Romanian maintenance instructions, and the two progressive curves, input or output of the circular curve, are not equal. For a rectification or retraction of the studied curves, which allows to maintain the designed radii or a radius close to the value of the designed one for the circular curve, it is necessary to increase the length of the progressive curves.

Taking in consideration that in most of the cases, it is required the use of longer progressive curves than the permissible ones for the cubic parabola transition curves, the retraction of those curves can be made by using another type of progressive curve, the improved cubic parabola or cosine progressive transition curve.

However, in case of transition with a cosine progressive curve, it must be considered that the cosine variation of the curve will also impose the same law of variation on the superelevation ramp.

References

[1] Bercu, M., Radu, C.: Transition curve for high speed train, Transition curve, pp. 27-51, 1968.

[2] Herman, A., Kazinczy, L., Köllő G.: Railway – Geometrical elements, Transition curve, pp. 55-72, 2011.

[3] Stoicescu, G.: Railway superstructure, Transition curve, pp. 90-106, 2009.

[4] Nechita, M., Köllő G.: Railway, Transition curve, Retraction of railway curves by angular diagram method, pp. 233-257, 1982.

[5] Hungarian National Railway, Railway construction and maintenance, Transition curve, 1 (2000), pp. 37-41

[6] Köllő, G.: Railway superstructure, Causes of railway track impairment, pp. 157-188, 1999.